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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
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MAY 1923

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VOLUME LVII MAY 1923 NUMBER 4

THE DURATION OF THE PROCESS OF LIGHT EMISSION IN HYDROGEN

By A. J. DEMPSTER

ABSTRACT

Duration of light emission from hydrogen canal rays.—Observations: Canal rays were allowed to pass through a small hole in the cathode into a second chamber at a very much lower pressure. With certain discharge conditions the rays ceased to be luminous in $1\frac{1}{2}$ to 4 cm; with others a strong continuous bundle masked the dying out phenomenon. By measuring the dependence of the intensity of the bundle on the pressure of the gas in the high vacuum chamber and simultaneously observing the Doppler effect, it is found that the continuous bundle is not due to residual luminosity, but to a second type of long-lived sources carried in from the discharge tube, whose radiation process lasts longer than 5×10^{-7} seconds. A strong electric field is found to have no effect either on the long-lived or short-lived rays. The dependence of intensity upon pressure shows that no rays are present that die out completely in a time much shorter than 10^{-7} seconds. *Interpretation.*—The explanation is proposed of the short-lived particles being neutral hydrogen atoms, and the continuous bundle being due to the dissociation of neutral molecules. *Theories of light emission.*—The classical radiation theory and Bohr's theory are discussed in connection with the above observations, and the observations of the Stark-effect in a variable electric field.

It has been observed by W. Wien[1] and by the author[2] that when a rapidly moving beam of hydrogen canal rays is allowed to pass into a high vacuum, the rays do not cease to give light at once but give a luminous bundle that dies out gradually in 2 to 4 cm. The natural interpretation of the phenomenon is that we observe the gradual decrease of the radiation from the moving sources in the rays and

[1] *Annalen der Physik*, **60**, 597, 1919; *ibid.*, **66**, 230, 1921.

[2] *Physical Review*, **15**, 138, 1920: Abstract of paper read at meeting of the Physical Society, November, 1919.

that after a certain time they are all extinguished. Professor Wien has measured the rate of decrease and finds it in approximate agreement with the value predicted by the classical electron theory for a single radiating atom. Experiments have also been made by W. Wien,[1] G. S. Fulcher,[2] J. Koenigsberg, and J. Kutschewski,[3] H. v. Dechend and W. Hammer[4] on the variation of the intensity of a canal ray bundle with gas pressure. With certain discharge conditions the intensity may be made very low by reducing the pressure of the gas through which the rays move.

The experiments reported in this paper extend the earlier observations and show that none of the rays die out at a rate faster than that observed by Wien and the author, and further that under certain discharge conditions a luminous bundle is formed that dies out at a much slower rate than that previously observed.

APPARATUS

The cathode and essential features of the discharge tube used in the first experiments are shown in Figure 1. The cathode, which had an aluminum face fastened to an iron back, rested on the ground end of a short glass tube sealed as shown, so that the only communication between the upper and lower chambers was a small hole in the center of the cathode 0.25 mm in diameter and 0.25 mm long. The upper chamber was connected to a mercury vapor pump and hydrogen was allowed to stream into the discharge tube. In this way the pressure in the lower chamber could be kept high enough for a discharge to pass while that in the upper was very much less. The canal rays were found to be luminous on first entering the upper chamber and to die out in a distance of 4 to 5 cm. Under the best conditions the upper part of the high vacuum tube was perfectly black and free from any general luminosity. Under certain con-

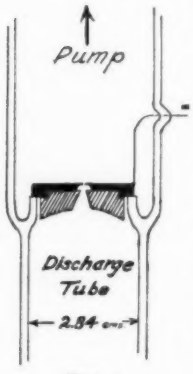

FIG 1

[1] *Annalen der Physik*, **30**, 349, 1909. [2] *Astrophysical Journal*, **33**, 32, 1911.

[3] *Sitzungsberichte der Heidelberger Akademie*, **1**, 4, 7, 1910.

[4] *Ibid.*, **1**, 21, 9, 1910.

ditions in addition to the decreasing luminosity, a bundle of uniform intensity could be observed continuing on to the end of the tube 50 cm beyond. This was at first attributed to the presence of sufficient gas in the upper chamber to re-excite a few of the canal rays and to be excited by them. Observation through colored screens and later with a direct vision prism showed that most of the visible light came from Hβ; Hα and Hγ could also be detected and appeared to die out at the same rate as Hβ.

Measurements of a photograph of the luminous bundle made with an ordinary camera and measured with a Hartmann microphotometer are given in Figure 2. The beam was thrown slightly out of focus so as to avoid any apparent decrease of intensity due to the slight divergence of the bundle. In the upper curve the scale readings of the instrument are given corresponding to different distances from the hole. The lower curve gives the background

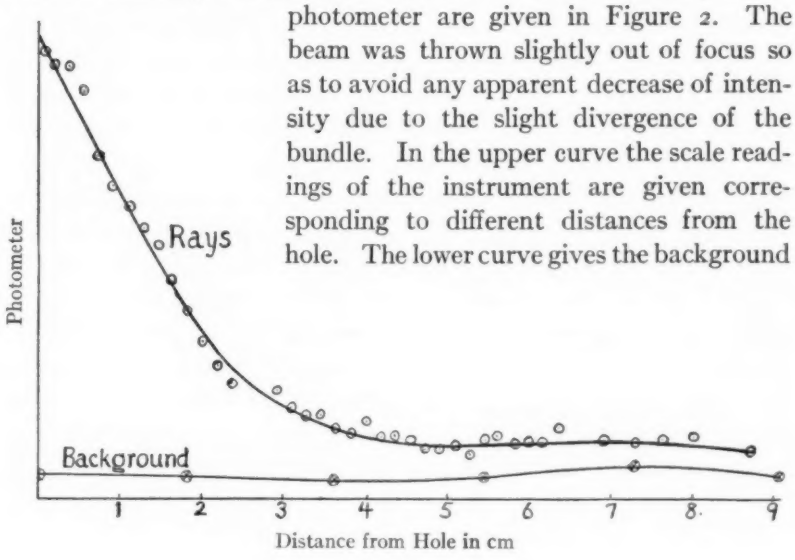

FIG. 2

reading, measured to one side of the bundle. It is seen that after 4 cm the intensity has fallen practically to a final steady value. Thus we may conclude that the fastest particles have died out in this distance. The speed of the rays may be obtained from Doppler effect measurements just before the cathode as approximately 4×10^7 cm per second, giving $4\div4\times10^7=10^{-7}$ seconds as an upper limit for the time in which this radiation process is completed.

In the abstract published in the *Physical Review*, **15**, 138, 1920, the velocity of the rays was taken as 8×10^7 cm per second from

observations of Wilsar and Stark and W. Steubing[1] on the Doppler effect in a beam behind the cathode. The Doppler effect just before the cathode is considerably smaller than behind, and from the fact that the rays enter the high vacuum almost immediately, it seems that the Doppler effect just before the cathode, viz., 4×10^7 cm per second, should be used.

INTENSITY CHANGES WITH PRESSURE

Further experiments were carried out to find the origin of the continuous bundle and to see if any rays were present with extremely short life. This was done by observing the changes produced in the intensity of the bundle by alterations of the pressure in the high vacuum chamber. It is known from the experiments of Fulcher[2] and Wilsar[3] that the sources of the undisplaced line in the Doppler effect are the gas molecules through which the flying particles move, while the sources of the displaced line are the flying particles that have come from the discharge tube, and are kept excited by the gas through which they move. The effect of reducing the pressure in the high vacuum chamber to zero is thus to suppress the undisplaced light from the gas, and to leave the flying particles free to die out. The luminosity from the moving sources is carried on into the high vacuum chamber and *the initial intensity of the bundle should be reduced only in the ratio of the intensity of the displaced component in the Doppler effect to the total intensity in both the undisplaced and displaced lines.* If the canal ray bundle contained many particles that die out in less than a millimeter, the intensity 1 mm above the hole should be reduced by lowering the pressure in the upper chamber to a much lower value than that given by the above ratio. If the continuous bundle several centimeters above the hole is reduced to a very low value by reducing the pressure, it may be ascribed to residual luminosity, but if it is reduced only in the ratio given above, then it must be due to long-lived luminous sources from the discharge tube.

[1] *Annalen der Physik*, **28**, 994, 1909.

[2] *Astrophysical Journal*, **35**, 101, 1912. [3] *Annalen der Physik*, **39**, 1295, 1912.

EXPERIMENTAL ARRANGEMENT

The first experiments on the alteration of the intensity in the upper chamber with pressure gave very uncertain results, probably due to the fact that the intensity of the Doppler effect, or proportion of moving sources in the rays, varies greatly with the purity of the gas and also with the discharge potentials.

The cathode in Figure 3 was therefore designed in order to observe the Doppler effect at the same time as the changes in intensity were measured. The iron cylinder was 4.4 cm long, had an aluminum face with a hole $1\frac{1}{2} \times 6$ mm, and rested on the polished end of a short glass tube. A window *W* was ground into one side and the Doppler effect in the rays could be observed through it and the prism *P*. A spectroscope with two large Rutherford prisms was used and the displacement at Hβ could be easily followed. Wilsar has shown that the Doppler effect does not alter to any extent with distance behind the cathode, so that the displacements observed may be taken as indicating the velocity of the rays after they have passed into the upper chamber. The small hole in the plate at the top was $\frac{1}{4}$ mm in diameter and 2 mm in length. Other openings were tried, a slit in a thin plate and a hole in a disk, but these gave qualitatively the same results as the hole in the thick plate and did not allow of such great pressure alterations. Two millimeters above the hole a short condenser was arranged to see if the luminous rays were charged or uncharged and to observe whether there was any effect of an electric field in altering the rate of dying out. The pressures were measured on a McLeod gauge, and it was found that a ratio of more than 100 to 1

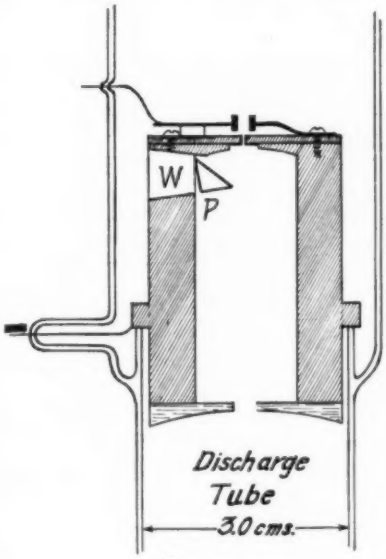

FIG. 3

could be maintained on the two sides of the cathode. The brightness of the bundle at various points was measured by matching its intensity with that of the real image of a small part of a piece of milk glass that was formed inside the tube directly beside the bundle. The image and bundle were viewed through a lens and the color was made as close as possible by colored screens. The intensity of the image was altered by varying the current through a lamp that illuminated the milk glass, and the current readings were later reduced to intensities by means of photometer comparisons.

The discharge tube was run sometimes from an induction coil with a mercury interrupter, but usually from a high voltage alternating current, rectified by a kenotron as used in X-ray equipments. In this way currents up to 10 milliamperes could be sent through the tube at most pressures.

DEPENDENCE OF DYING OUT ON DISCHARGE CONDITIONS

With the cathode in Figure 3 it was found that the dying out was prominent only with high pressures and low voltages in the discharge tube, even when the pressure in the upper chamber was less than 1 per cent of that in the lower. The dying out was observed with pressures in the discharge tube between 0.44 mm and 0.16 mm, voltages 2500–3500; currents up to 9 milliamperes. With the low pressures larger currents were necessary in order to make the dying out effect distinct, as otherwise, with small currents, a continuous bundle was superimposed. Using large currents of 5–10 milliamperes, the Doppler effect appeared suddenly as the pressure was reduced, accompanying an alteration in the discharge conditions before the cathode, and the sudden appearance of the short bundle in the upper chamber, which was previously completely black, was simultaneous with this alteration. The bundle seemed to lengthen from $1\frac{1}{2}$ to about 4 cm as the Doppler displacement increased, with reducing pressure. When the pressure in the discharge tube was reduced to about 0.13 mm, depending on the currents used, a faint continuation of the bundle was observed; and at a slightly lower pressure of 0.12 mm, with a voltage of 5000 and a current of 9.5 milliamperes, the continuation became so

intense that the dying out in the first few centimeters was almost completely masked by the stronger uniform bundle.

As the pressure was reduced still further, for instance to 0.072 mm (voltage 15,000, current 2 milliamperes), the continuous bundle remained very strong in comparison with the faint bundle previously observed and no decrease in intensity in the first few centimeters could be observed. The Doppler effect at this pressure was, however, still very distinct, the displaced line being approximately equal in total intensity to the rest line.

THE EXISTENCE OF LONG-LIVED RADIATING SOURCES

The continuous bundle was at first attributed to the residual gas in the upper chamber which would be excited by the beam of rays passing through it. The decrease in the distinctness of the dying out phenomenon could be associated on this hypothesis with the fact that at high voltages the displaced intensity in the Doppler effect may become very small compared with the intensity in the rest line[1] (Fig. 4) so that we might expect very few luminous particles to enter the upper chamber.

The simultaneous observation of the Doppler effect made this hypothesis untenable and showed that the strong continuous bundle appears before the Doppler effect has become faint. This suggested that the luminosity might be due to moving sources from the discharge tube that have a very much slower rate of dying out than those appearing at higher pressures.

In the earlier experiments with the short cathode the continuous bundle was not so prominent even at low pressures as in the cathode in Figure 3. This is possibly connected with the fact that with the short cathode in Figure 1 the rays make very few collisions before entering the upper chamber, and suggests that the long-lived radiators may be formed as a result of collisions. The great increase in the Doppler effect just behind the cathode over that just in front[2] shows that a few collisions of the moving particles

[1] F. Paschen, *Annalen der Physik*, **23**, 260, 1907; B. Strasser, *ibid.*, **31**, 916, 1910.

[2] B. Strasser and M. Wien, *Physikalische Zeitschrift*, **7**, 746, 1906; W. Wilsar, *Annalen der Physik*, **39**, 1291 and 1295, 1912; F. Paschen, *Physikalische Zeitschrift*, **7**, 924, 1906; J. Stark, *ibid.*, **7**, 747, 1906.

cause a great increase in their luminosity. That the continuous bundle was largely due to moving sources from the discharge tube was proved to be the case by measuring the dependence of the intensity on the pressure in the second chamber while the conditions in the discharge tube were kept unaltered. If the continuous bundle is an excited luminosity in the residual gas, its intensity should be much reduced if the gas pressure in the second chamber is reduced to a low value, while if its intensity is not reduced appreciably the luminosity must be carried in by sources from the discharge tube.

Observations of the intensity of the bundle in the upper chamber about 3 cm above the hole are given in Table I. The first column

TABLE I

Pressure in Discharge Tube	p_1, First Pressure in Upper Tube	p_2, Second Pressure in Upper Tube	Pressure Decrease	Intensity Decrease	Volts on Discharge Tube	Current in Milliamperes	Length of Dark Space	Doppler Effect	Dying Out in First 4 cm
							cm		
.143	.0263	.00126	20.8	>10.0	3000	5.5	1.6	B	Plain
.127	.0145	.0015	9.6	4.0	5000	9.5	2.3	B	Doubtful
.094	.0233	.0015	15.4	1.43	7000	8	2.7	C	Absent
.075	.0158	.00075	21.1	2.1	10,500	5.5	4.0	C	Absent
.0725	.0145	.00067	21.6	3.25	15,200	1.7	4.2	D	Absent
.057	.0123	.0005	24.4	3.4	20,000	.....	.....	D	Absent
.0747	.0206	.00067	30.8	2.3	11,500	6.2	3.7	C	Absent
.16	.021	.00126	16.7	1.43	3250	8.5	.....	B	Plain

gives the pressure in the discharge tube in millimeters of mercury, the second and third the two pressures in the upper tube at which the intensity of the bundle was measured. The pressure change could be made by removing the heater from the mercury vapor pump till the pressure in the upper chamber had increased to the value given by p_1, when a general luminosity began to appear in the gas around the bundle. The heater was then replaced and the intensity observed. It was found that the pump began to act again suddenly, and reduced the pressure in the upper chamber from p_1 to p_2 in a few seconds without altering the pressure in the discharge tube. The fifth column gives the intensity change produced

by the pressure change in the fourth. The eighth column gives the distance in centimeters from the cathode to the end of the Crookes' dark space. The letters under Doppler effect refer to the sketches in Figure 4, which indicate the way in which the appearance of Hβ alters with increasing potentials on the tube.

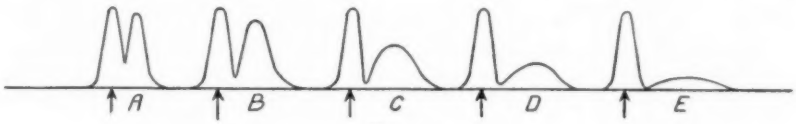

FIG. 4

The first experiment in the table refers to a condition in which the continuous bundle is practically absent, only the faint decreasing bundle appearing. With p_2 the intensity 3 mm above the hole has died down to a very small value, with p_1 there is a continuation of the luminosity. These conditions are similar to those in Fulcher's[1] experiments in which the intensity of the beam at a considerable distance in the high vacuum chamber was found to be proportional to the pressure. In the second experiment, the dying out is almost masked by the continuous bundle; in the third, fourth, and fifth, there is no indication of an initial dying out and the continuous bundle is very strong compared with that in the second. The slight alteration in intensity caused by a pressure change of more than 20 times proves, as discussed above, that the luminosity cannot be excited in the residual gas but must be carried in by luminous sources. In the sixth experiment the Doppler effect has become relatively weaker, and there is in consequence a greater intensity change.

Since no change in the continuous bundle could be observed in 30 cm, we must conclude that under certain conditions particles of a second type are present in the canal rays with a luminous life greater than $30/6\times10^{7}=5\times10^{-7}$ seconds.

The last two rows in the table refer to measurements of the intensity of the bundle one millimeter above the hole. Here there is also a small intensity decrease produced by a large pressure

[1] *Astrophysical Journal*, **33**, 32, 1911.

decrease, and we must therefore conclude that no considerable proportion of the rays die out in less than one millimeter.[1]

NATURE OF THE PARTICLES WITH LONG AND SHORT RADIATION TIMES

In order to test whether the luminous sources were charged or uncharged a condenser was arranged, as shown in Figure 3, two millimeters above the hole. No effect of an electric field could be observed with either the strong continuous bundle or the bundle that disappeared after 4 cm, even when one plate of the condenser was connected to the anode. We must conclude then that in hydrogen both kinds of sources are in the neutral state. In a recent paper W. Wien[2] has also concluded that the luminosity in the disappearing bundle comes from neutral particles. We may also conclude that a strong electric field, such as used by Stark[3] in observing the electrical separation of spectrum lines, does not cause the extinction of luminosity previously excited in the sources. The observation of the Stark effect in the condenser is planned as a further investigation. On Bohr's theory the initial disturbed orbit would be produced in the lower chamber and correspond to zero field, while the final orbit is reached in a strong field. The observations may be expected to throw light on the various theories of the atomic processes discussed below.

No definite conclusion can be drawn from these experiments alone as to the nature of the two kinds of radiating particles. We might think of neutral atoms and neutral molecules, for both occur in the canal rays,[4] but since the radiation is always almost entirely

[1] In a recent article in the *Annalen der Physik* for January, 1923, Professor Wien develops a very complete theory of the dying-out phenomenon in connection with the changes of the charge on the flying sources. His experiments show that there is very little further change in the dying-out phenomenon when the pressure in the high-vacuum chamber is reduced below 0.005 mm. The pressures p_2 in Table I are below the values at which there can be any effect from the residual gas as calculated by Professor Wien from the various free paths occurring in the rays. In his notation we may say that the experiments in this paper show that there are two distinct values of the constant l, which indicates the distance covered by the rays before the radiation is emitted. In Professor Wien's observations of the dying-out phenomenon the potentials on the discharge tube are given as less than 5,000 volts, so that under his discharge conditions the long-lived sources were probably unimportant.

[2] *Annalen der Physik*, **69**, 325, 1922. [3] *Ibid.*, **43**, 968, 1914.

[4] J. J. Thomson, *Rays of Positive Electricity*, p. 67.

the hydrogen series lines we should endeavor to ascribe the light in both cases to atoms. We may provisionally adopt the explanation that the comparatively faint short bundle observed at high pressures and low voltages is due to atoms formed in the discharge tube that have become neutralized before entering the upper chamber and emit light in the neutral state. The time for the duration of the radiation previously observed by Wien and the author then applies to these neutral atoms. The sources that give rise to the strong continuous bundle may be supposed to be atoms formed by the relatively slow dissociation of neutral molecules that have come from the discharge tube. Experiments by G. P. Thomson[1] show that charged hydrogen atoms are more numerous at high pressures and charged molecules at low, and we may assume that the same applies to the neutral constituents of the rays. On this view the neutral hydrogen molecule either does not dissociate, or is not formed in the discharge tube with potentials below 3000 volts. With higher potentials, its collisions while still in the lower chamber cause some instability, resulting in dissociation in the upper chamber with a consequent emission of the hydrogen series lines. The lower limit for the time required for this radiation process, 5×10^{-7} secs., as deduced above, applies on this view to the rate of dissociation of instable hydrogen molecules.

INTERPRETATION IN TERMS OF ATOMIC PROCESSES

The observed duration of the radiation from a moving beam of canal rays may be interpreted in terms of the atomic process in several ways. In Wien's earlier papers a mean value for the velocity is used, and the exponential intensity decrease observed is supposed to be the sum of exactly similar intensity decreases for the radiation from individual flying atoms. This interpretation is in accord with that required by the classical theory of continuous radiation from an electronic oscillator and the observed value for the damping coefficient also agrees well with the numerical value predicted on this theory.[2] Objections to this apparent con-

[1] *Philosophical Magazine*, **40**, 242, 1920.

[2] H. A. Lorentz, *Theory of Electrons*, p. 260.

firmation of the simple radiation picture were raised by Seeliger,[1] who pointed out that the particles are excited at various distances in front of the hole. I have also found that if we assume a distribution of velocities, similar to that shown by Doppler effect curves, instead of a single velocity as assumed in Wien's calculations, the observed intensity decrease may even be obtained from particles that emit at a uniform rate and cease suddenly. Such a type of radiation process was discussed in a paper by Epstein.[2] Recently Mie[3] has developed a theoretical curve for the decrease, from consideration of the phenomena to be expected with Bohr's atom model.

A second interpretation may be called, on Bohr's picture of the radiation process, the theory of delayed jumps—the displaced electron may remain for some time in an outer non-radiating orbit and the duration of the actual light emission may be comparatively brief. Thus the pencil of decreasing luminosity observed in the second chamber may be made up of individual flashes or short streaks of light emitted when the electron does return to the inner orbit. The curve in Figure 2 is thus to be regarded as a decay curve for the semi-stable excited atoms formed in the discharge tube, analogous to the exponential decay curves of radioactive substances. This "quantum" interpretation has been discussed by O. Stern and M. Vollmer[4] in relation to a paper by Einstein.[5]

A third interpretation is the dissociation theory suggested above to explain the continuous bundle. Here we think of the emission process as following a dissociation process.

STARK EFFECT OBSERVATIONS AND DURATION OF THE EMISSION

A discussion of Stark's experiments on the effect of an electric field on spectrum lines, and their bearing on the duration of the emission process has recently been published by Foersterling.[6]

[1] *Jahrbuch der Radioaktivität*, **16**, 420, 1920.

[2] *Münchener Berichte*, p. 73, 1919.

[3] *Annalen der Physik*, **66**, 237, 1921.

[4] *Physikalische Zeitschrift*, **20**, 183, 1919.

[5] *Ibid.*, **18**, 123, 1917. [6] *Zeitschrift für Physik*, **10**, 357, 1922.

In Stark's[1] discharge tube the flying luminous particles move through holes in the cathode into a strong electric field. If the radiation from an atom lasts 10^{-7} seconds as deduced above, the sources will move 1–4 cm during the process, and should carry their light into the electric field.

Stark states that under the conditions in his discharge tube most of the light must have come from the moving particles and adopts precautions to avoid the Doppler effect from their motion. In Stark's experiments the dark space was 5 to 10 centimeters in length and it is probable that the gas pressure was less than 0.05 mm of mercury, giving a mean free path greater than 1.5 cm. This according to Wien's experiments is also approximately the free path for changes of sign in the canal rays, so that the radiating particles should move undisturbed for about this distance. We have shown above that their luminosity is not affected in intensity by a strong electric field. His observations, however, show no unaffected lines in the field with Hα, Hγ, and Hδ (p-components). Foersterling concludes that the radiation from an atom must be capable of being altered during the process of emission if the atom moves into an electric field. He proposes the idea of some continuous connection between the atom and the energy to be radiated that would allow of such continuous alterations in frequency as are observed, and would abandon Sommerfeld's idea of the energy radiation process in itself being distinct from the releasing of the energy in the atom.

If, however, we are to keep the fundamental idea of Bohr's theory that the frequency radiated is determined by the energy difference between two orbits, the following considerations lead us to suppose that the elementary radiation process is completed in a very brief time. In his 1915 paper Stark observes that the field between the cathode and the secondary electrode has a very strong gradient over about 0.2 cm near the cathode. The lines are split up in this varying field so that the displacements follow sharply the variations in the field strength. These observations are predicted with remarkable detail by the theory of Epstein on the basis of a calculation of the modifications introduced by an electrical

[1] *Annalen der Physik*, **43**, 968, 1914; **48**, 194, 1915.

field in the Bohr non-radiating orbits. The significant point is that the final as well as the initial modified orbit determines the frequency of the light emitted. If in the varying part of the field a frequency characteristic of one field intensity is emitted, the electron must have passed from the initial to the final orbit while the flying atom has covered less than 0.2 mm. We are also bound to say that the energy difference must have been completely radiated in the same short interval. The only logical alternative would be for the atom to act in some way as a receiver of energy and to emit it later in such a manner that the frequency is altered to suit the new field strengths existing at each instant. This alternative suggested by Foersterling makes the frequency radiated independent of any orbital considerations whatsoever, and so removes the basis of Epstein's theory and the Bohr theory of light emission. The successes of these theories, however, should, at least, be considered as confirming Bohr's idea of connecting radiation frequencies with differences in orbital energies.

We are thus led to the conclusion that the whole emission process on the Bohr picture, including the transition between two stationary orbits, and the radiation of the energy difference, must occur while a flying atom with a speed of approximately 6×10^7 cm per second covers 0.2 mm; that is, in less than 3×10^{-10} seconds.

The success of Holtsmark's[1] theory of the broadening of spectral lines also gives indirect evidence on this point. The broadening is supposed to be due to the Stark effects produced in the radiating atoms by the superposition of the electric fields from other flying atoms. This resultant field varies very rapidly and the radiation emitted has a frequency corresponding to the field that happens to exist at the particular instant of the light emission.

It is of interest to consider whether the three interpretations of the canal ray experiments, discussed above, are compatible with the very brief duration of the elementary atomic radiation process, as deduced from Stark's experiments. The first classical interpretation gives too long a time as pointed out by Foersterling. The second interpretation of the duration of the luminosity as due to "delayed jumps" might agree as far as the brevity of the actual

[1] *Annalen der Physik*, **58**, 577, 1919.

emission is concerned, and may apply to the faint short-lived bundle, but it is impossible to have "delayed jumps" in Stark's experiments, since it is necessary to have the initial and final orbits both characteristic of some definite electric field. The third explanation proposed above to explain the long-lived continuous bundle, in which the emission from an atom is regarded as following the dissociation of a molecule, may be reconciled with Stark's observations, since it seems probable that most of the moving intensity in Stark's experiments was of this kind, inasmuch as the pressure in his tube was such as to give a dark space 5 to 10 cm in length. Here the initial orbit, radiation emitted, and final orbit may all be characteristic of the point in the field at which the dissociation of the molecule leads to light emission.

Although we may thus obtain agreement between Bohr's theory and canal ray experiments, by supposing a duration of the light emission process of less than 3×10^{-10} seconds, we are led thereby to disagreement with the observed homogeneity of spectrum lines. The value for the half width for Hα, .047 A, as observed by Michelson[1], agrees with the value computed from the Doppler effect caused by the kinetic motions of hydrogen atoms if the temperature is assumed to be 135° C. The finite wave train of 9 cm caused by a limitation of the time for the emission to 3×10^{-10} seconds would cause a half width of at least .01 A.[2] This would leave only .037° A as due to Doppler effect and this is smaller than that demanded by the kinetic theory even if the tube were at room temperature. With helium a very great discrepancy may be deduced.

SUMMARY

The experimental results may be summarized as follows:

1. The previously reported experiments on the dying out of a beam of moving luminous sources in hydrogen canal rays in 2–4 cm have been repeated, and the conditions investigated under which the dying out is most prominent. The upper limit for the time of duration of this radiation process is 10^{-7} seconds.

2. No luminous rays are present that die out in a much shorter distance.

[1] *Philosophical Magazine* (5) **34**, 280, 1892. [2] *Astrophysical Journal*, **2**, 252, 1895.

3. With different discharge conditions a second type of moving luminous particles appear that show no dying out. Their radiation process lasts longer than 5×10^{-7} seconds. With low pressures they are much more intense than the short-lived particles.

4. Both types of luminous particles are neutral, and their luminosity is not altered in intensity by a strong electric field.

The discussion of the experiments in connection with other phenomena bearing on light emission shows that the first type of particles may be neutral hydrogen atoms. The second type of luminosity may be a secondary phenomenon following the dissociation of a neutral molecule. Stark effect observations in a variable electric field lead us to assume that the duration of the elementary atomic radiation process requires less than 3×10^{-10} seconds. This short time may be reconciled with one interpretation of the experiments in this paper, but leads to disagreement with the duration of the atomic radiation set by the classical interpretation of the homogeneity of spectrum lines.

The author's thanks are due to Professor Frost and Professor Parkhurst, of the Yerkes Observatory, for the use of their Hartmann microphotometer, and to Dr. F. Kannenstine for many mechanical suggestions.

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A STUDY OF THE GREEN AURORAL LINE BY THE INTERFERENCE METHOD[1]

By HAROLD D. BABCOCK

ABSTRACT

Green auroral line; observations with Fabry-Perot etalon.—It is found that with a camera having a speed of $f/3$ or more the persistent green radiation in the background of the night sky can be observed through a Fabry-Perot interferometer on almost any night between third and first quarters of the moon. No dispersion apparatus is required. Moderate cloudiness does not hinder the observations. Orders of interference amounting to 3700, 8450, 15,500, 31,000, and 85,000 waves were successively used in photographing the rings and from the sharpness it is concluded that the *line-width* is not more than 0.035 A. No appearance of doubling or other complexity has been noted. The surface *brightness* of the diffuse green auroral light is found to be of the order of 10^{-8} times that of the green radiation from a mercury arc in vacuum. Considerable variations in the intensity have been noted, probably amounting to at least three- or fourfold. A consistent series of eleven interferometer measurements of the *wave-length* is given. Two pairs of planes, three different separations, and three different pairs of metal films, both gold and silver, were used. The green mercury line and the strongest yellow neon line were used as reference standards. The wave-length obtained is $5577.350 \pm .005$ I.A.

INTRODUCTION

An extensive list of papers gives evidence of the interest which investigators have shown in the spectrum of the polar light. While the beautiful researches of Störmer, Birkeland, Vegard, and others have contributed greatly to our knowledge of the nature and causes of the aurora, the origin of the prominent green line characteristic of this light is still unexplained.

It has long been recognized that on nights when no auroral display is visible there still may be observed in the background of the sky a faint yellow-green spectral line which has been identified with the strong line in the spectrum of the aurora itself. The importance of further study of this light is emphasized by several recent investigations. Lord Rayleigh[2] has shown that the green line can be photographed in England on two nights out of three, even when the sky is partially clouded. He also found that it was more intense at Terling near London than some three degrees of latitude farther north, near Newcastle. Slipher[3] has published the results of

[1] *Contributions from the Mount Wilson Observatory*, No. 259.

[2] *Proceedings of the Royal Society*, A, **100**, 367, 1922.

[3] *Astrophysical Journal*, **49**, 266, 1919.

photographic determinations of the wave-length of the green line made with higher dispersion than had been employed previously. He found the wave-length to be 5578.05 A from three plates made with exposures of about 100 hours each. The investigation of Vegard[1] leaves no question as to the identity of the persistent green line with that present in the spectrum of visible streamers and discusses the difficulties of accounting for it, as well as the possible manner of its excitation.

The extreme faintness of this persistent light, on the one hand, and the evanescent character of most other auroral phenomena, on the other, have hitherto restricted observers to the use of low-dispersion prismatic instruments for such investigations.

It is the purpose of the present paper to describe work done in Pasadena and on Mount Wilson during recent months without the customary dispersing apparatus, and to present the results of new measurements of the wave-length of the green line made by the interference method, together with some observations on its width and intensity.

Inasmuch as this line is the only emission line found within a long range of the spectrum of the night sky, it should, if sufficiently homogeneous, give interference rings when a Fabry-Perot etalon is placed in front of a camera focused for parallel light and pointed toward the sky. Superposed light, having either a continuous or a band spectrum, would spread a uniform illumination upon the ring system, and the visibility of the latter would depend not only upon the sharpness of the spectral line but also upon its brightness relative to the background. The available data upon the brightness made it seem probable that under good observing conditions the background illumination would not be a very serious obstacle, and this has proved to be the case.

LINE WIDTH AND INTENSITY

An etalon having glass planes 1 mm apart was mounted in front of a hand magnifier lens of about 50 mm equivalent focal length and 25 mm clear aperture. The planes were covered with very thin

[1] *Geofysiske Publikationer*, 2, No. 5, "Utgit av den Geofysiski Kommission"; *Philosophical Magazine*, 42, 47, 1921.

films of gold cathodically deposited, this metal being chosen because of its high coefficient of transmission for green light. The first plate was exposed for ten hours in Pasadena on the night of February 25–26, 1922, and upon development the clear impression of a system of circles was found. No attempt was made to maintain the temperature constant during the exposure and no means were employed for reducing the background illumination. The instrument, packed in a wooden box, was merely directed toward a point low in the northern sky, away from the glare of the city lights,and left throughout the night with an alarm clock set to close the shutter before the first light of dawn. A photographic emulsion prepared by Mr. C. E. K. Mees of the Eastman Kodak Company was used. This is called an "Astronomical Plate" and has a uniformly high sensitivity throughout the green region of the spectrum. Several other emulsions tried later were found less rapid.

Instrumental improvements were at once effected, and it was soon found that the first exposure was not exceptional. On the contrary, almost every plate exposed under fair conditions has shown at least some trace of the interference ring system, and even on partially cloudy nights good photographs have been obtained.

Without relating in detail all of the trials made, it will prove sufficient to describe here the instrument as used in the later stages of the work. The camera is supplied with a Dallmeyer cinematograph anastigmat lens of aperture ratio $f/1.9$, having an equivalent focal length of 76.8 mm. It has a spiral focusing mount and is rigidly fastened in a light metal box, provided with a brass plateholder. The interferometer is supported upon the same aluminum base plate, close in front of the camera lens. It consists of a pair of planes of either glass or fused quartz, held parallel to one another by an invar separator, the system being contained in a steel tube carrying the adjusting springs at one end. A cover of sheet aluminum incloses both etalon and camera, and the combination may be quite freely handled without disturbing the adjustment. The clear aperture of the interferometer plates when in position is 25 mm, thus reducing the effective aperture of the camera to about $f/3$. The instrument is mounted in a wooden case which is provided with an extension 1.5 meters long in front. A piece of plate glass

closes the opening, and a simple sliding shutter is actuated by a cord wound on the spindle of the ringing mechanism of an ordinary alarm clock. The interior of the compartment containing the apparatus is provided with a thermostat of bimetallic strip and a distributed heating resistance. The entire case is packed in wool in a much larger wooden box. These precautions make it possible to reduce the variations of temperature to less than 0°.1 C. during an exposure.

It was found that the 1 mm etalon gave sharp fringes. At this point, the following question might be raised: Is it certain that the fringes photographed are produced by the green line? An examination of the negatives under a measuring machine shows that the scale of the ring system itself provides an affirmative answer, for a rough value of the wave-length is given by the expression,

$$\lambda = \frac{2eS}{8F^2}$$

where e = thickness of etalon; F = equivalent focal length of camera; S = difference in squares of successive ring diameters. A measurement of S and its probable error upon a plate taken with the 1-mm etalon, together with the known values of e and F, gives for the wave-length the value 5525 ± 70 A. The difference between this approximation and the known value of the wave-length is thus well within the probable error. Observations to be described later remove all uncertainty in this regard.

The etalon plates were next separated more widely. The second distance chosen was 2.4 mm and no difficulty was experienced in securing sharp fringes with this order of interference, namely, about 8450 waves. The next higher order employed was 15,500 waves, which was obtained by means of a plane-parallel glass plate of suitable thickness and index of refraction, lightly gilded on both sides. Sharp fringes were photographed with this instrument.

The fused quartz plates referred to above were then separated about 8.7 mm by pieces of invar and subsequent photographs, at about 31,000 waves retardation, still showed sharp rings. A thicker plate of plane-parallel glass was then used, giving an order of interference for the green line of approximately 85,000 waves. With

this high order of interference the rings are very small, but they are still well defined and a conservative estimate of their width places this quantity at not over one-half of the spectral range of the etalon, which is 0.066 A. It would appear safe to say that the width of the persistent green auroral line does not exceed 0.035 A. This is approximately the width of the finer arc lines of iron when excited in a vacuum by a moderate electrical current.

It may be mentioned here that the spectral range, or the extent of spectrum contained between any two points in the interference pattern whose orders of interference differ by unity, is given by the expression

$$\text{Spectral range} = \frac{\lambda}{p} = \frac{\lambda^2}{2en}$$

where λ = wave-length; p = order of interference; e = thickness of etalon; n = index of refraction of the medium between the parallel planes.

None of the photographs obtained show any signs of doubling or dissymmetry in the line; the number of different orders of interference employed, together with the fortuitous values of their ratios, would indicate that the line is either single or a doublet having a separation less than 0.035 A, or, if it consists of a primary and satellites, the latter must be of low intensity.

With these high orders of interference and the short-focus camera, the ring systems are on so small a scale that further large increase of resolving power is impracticable. To increase the scale by the use of a longer-focus lens requires an etalon of proportionately greater diameter, if one is to maintain sufficient photographic speed to secure an observation in a single night. A larger etalon is now under construction in the instrument shop of the observatory with which still higher resolving power may be brought to bear upon the green line while still permitting the use of very rapid camera lenses.

Most of the photographs discussed here were made when the moon was below the horizon throughout all or nearly all of the exposure time. Under this condition it is found that, even in Pasadena, fair contrast may be secured between the rings and the back-

ground without the use of a color screen, except when clouds or fogs diffuse the light from street lamps. On Mount Wilson no trouble arises from artificial illumination if the northern sky is used, and, even when the moon is at the quarter phase, good plates may be obtained if a filter is used to cut off light of wave-length less than about 5000 A.

The best screen is found to be one designed for isolating the green line of mercury from the other lines in the spectrum of that element. The intense absorption bands of neodymium in the yellow and the extinction of the entire spectrum below λ 5200 leave only a small portion of the continuous spectrum effective, since the plates used are comparatively insensitive to red light. The amount of background fog is thus appreciably reduced.

When all observing conditions are as uniform as possible there still appear large differences of intensity among the photographs secured. It would seem that variations in brightness of perhaps three- or fourfold are of frequent occurrence. At times, the intensity was found exceptionally high, the best instances of this being the observations of July 11–12 and July 12–13. There appeared concurrently at the sun's limb a remarkable prominence which subsequently extended over one-fourth of the solar circumference, and this event was probably associated with the increase of intensity noted. Since the present year is near a minimum in the cycle of sun-spot activity, it would not be expected to offer the best opportunities for a study of this kind, although, if systematic observations had been made, some further correlation between intensity of persistent auroral light and solar phenomena might have been revealed.

A consideration of the data regarding exposures made upon the aurora and the green mercury line leads to an estimate of the order of magnitude of the ratio of the surface brightnesses. The mercury lamp was inclosed in a box having an opening covered by the filter which transmitted only the green radiation. This light fell upon a white screen having a good diffusing surface, which acted as a reflector and furnished light to the etalon and camera. The area of the luminous mercury vapor used was about 3 square centimeters, and it was distant approximately 60 centimeters from the diffusing

screen. The reduction of surface brightness is then found by comparing the area of a hemisphere of radius 60 cm with the area of the source. This ratio of areas is about 7000:1. The average exposure time for the aurora was about nine hours and for the mercury light about five seconds, but, on account of slower plates being used for the mercury, it is fair to take the corrected ratio of exposure times as about equal to 10^4. Since the auroral plates were generally weaker than the comparison photographs, it follows that in round numbers the ratio of the surface brightness of the green radiation from a mercury arc to the surface brightness of the diffuse auroral light is about 10^8. A promising field is offered here for more extended quantitative work upon the brightness of the green auroral light and its variations.

It is worthy of note that, for studies of the intensity of monochromatic sources such as the green auroral line, instruments like the one described offer certain advantages. This is especially true when the etalon consists of a plane-parallel glass plate with gilded planes, for such a plate is always in adjustment if reasonably protected from temperature variations, and its gold films are fairly permanent. If need be, the metal films can be protected by glass covers, one of which may be a color screen, cemented on with Canada balsam. Results comparable with those of other observers may be obtained if the camera speeds are specified and the absorption factor is measured. Since a moderate order of interference is in general best suited to intensity observations, there is required a plane-parallel glass plate of sufficient size which is not too thick. But extreme accuracy in the surfaces is not essential, except when wave-length measurements are to be made upon the rings, and no great difficulty is found in preparing suitable plates.

It may be remarked that a plane-parallel glass plate affords a distinct advantage over an air etalon giving the same order of interference when used with short-focus lenses in the study of widths of spectral lines. It may easily be shown that, for the same value of focal length, and the same integral and fractional orders of interference, the diameter of a ring is directly proportional to the index of refraction of the medium between the reflecting planes. Thus, the use of a glass plate is equivalent to an increase of focal length

in the ratio 3:2 without loss of speed, spectral range, or resolving power. This increase of scale is very valuable when the limiting order of interference is approached.

The exposure times were dependent upon the hours of darkness; in each case, all the available time of a single night was used. Near summer solstice this amounts to only six hours at Mount Wilson, since exposures were not made when the sun was less than 18° below the horizon. No observations were made in June, but in July good plates were secured with about seven hours' exposures.

WAVE-LENGTH MEASUREMENTS

Turning now to the question of the wave-length of the green line, it may be recalled that the recent measurements of Slipher[1] and of Vegard[2] are of outstanding weight in comparison to those made previously. An uncertainty of 1 A in the values obtained by these observers is improbable. On this account it was deemed sufficient to begin measurements upon interference plates taken with an etalon having a spectral range of about 1.5 A, corresponding to a distance of 1 mm between the planes. The practice was adopted of photographing the green line of mercury through the instrument before and after each exposure to the aurora, always diffusing the light from a large surface in order to make the illumination of the etalon similar in the two cases. At times, the bright yellow line of neon λ 5852.488 was used in addition to the green line of mercury. This was conveniently supplied by a 5-watt Osglim lamp with a specially made filter to reduce the intensities of the other lines in the spectrum. The spectra of mercury, iron, and neon were used for determining the integral orders of interference.

As indicated above, there is no ambiguity attached to the wave-length derived from the 1-mm etalon if the uncertainty of the preliminary value is appreciably less than the spectral range. This condition is more than fulfilled in the present instance by the agreement between the measurements of Slipher and Vegard. But the use of two other widely different orders of interference furnishes an independent check. Since none of the orders employed bears a simple multiple relation to the others, the consistent values of the

[1] *Loc. cit.* [2] *Loc. cit.*

wave-length yielded by the three etalons can only arise from a unique system of corresponding whole orders, unless a preliminary wave-length, differing by more than 12 A from the known value, is adopted. This may be seen from an inspection of the spectral ranges of the various etalons, which are very approximately 1.535, 0.652, and 0.178 A, respectively, at wave-length 5577. The lowest integers having the same ratios as these numbers are 8, 19, and 69, and $8 \times 1.535 = 12.3$ A. Such an error in the observations of previous investigators is inadmissible.

In Table I are presented the results derived from all the plates thus far obtained which are suitable for wave-length determinations.

TABLE I

WAVE-LENGTH OBSERVATIONS ON GREEN AURORAL LINE

Date	Station	Reference Line	Etalon	Observed λ	Weight
			mm	I.A.	
1922, Sept. 20–21...	Mt. Wilson	5460.746	8.7	5577.348	2
Sept. 21–22...	Mt. Wilson	5460.746	8.7	.356	1
Sept. 27–28...	Mt. Wilson	5460.746	8.7	.353	1
Oct. 18–19...	Mt. Wilson	5460.746 5852.488	8.7	.344	5
Oct. 25–26...	Mt. Wilson	5460.746 5852.488	2.4	.346	3
Nov. 16–17...	Pasadena	5460.746	1.0	.372	1
Nov. 17–18...	Pasadena	5460.746	1.0	.355	1
Nov. 20–21...	Pasadena	5460.746	2.4	.353	1
Nov. 22–23...	Pasadena	5460.746	2.4	.350	1
Nov. 24–25...	Pasadena	5460.746	8.7	.352	2
Nov. 25–26...	Pasadena	5460.746	8.7	.352	2
Weighted mean.	.....	.....	.....	5577.350	.....

The weights given in the last column are based upon the difficulty of measurement and the degree of constancy of the etalon between comparison plates. Under the best observing conditions the change in thickness of the etalon during the night was found to be so small that if either comparison plate had been used without the other the error introduced would not have exceeded 0.002 A.

The values of the wave-lengths of the reference lines are those given by Fabry and Buisson[1] and the Bureau of Standards[2] for the

[1] *Journal de Physique* (5th series), **9**, 189, 1919.

[2] *Scientific Papers of the Bureau of Standards*, No. 329, 1918.

mercury and the neon lines, respectively, in terms of the primary standard of wave-length. The numbers in the column headed "Observed λ" are uncorrected either for change of phase in the interferometer or for dispersion of the atmosphere, due to the observations not being made under standard conditions, since an examination of the magnitudes of these corrections showed them to be small compared to the variations found. Two different pairs of gold films were employed on the fused quartz plates, one of these pairs of films having a small admixture of silver in the gold, and the observation of November 22–23 was made with pure silver films on glass plates. It seems probable that errors due to the imperfections of the interferometer and its adjustment are nearly eliminated when the separate values are combined, and it is thought that the weighted mean derived from the observations is reliable to approximately one part in one million. This mean is

$$\lambda = 5577.350 \pm 0.001 \text{ I.A.}$$

the probable error being appended to show the accordance of the data rather than to indicate the limit to the uncertainty in the wave-length.

This value of the wave-length is 0.48 A less than that of Slipher, and 0.63 A less than the measurement of Vegard, assuming that these observers refer their results to the Rowland scale, which seems to be implied in their papers. From an extensive study of the differences between the Rowland and the International scales made at this observatory, it is found that at λ 5577 the latter is 0.220 A below the former; and this correction has accordingly been applied before making the foregoing comparisons.

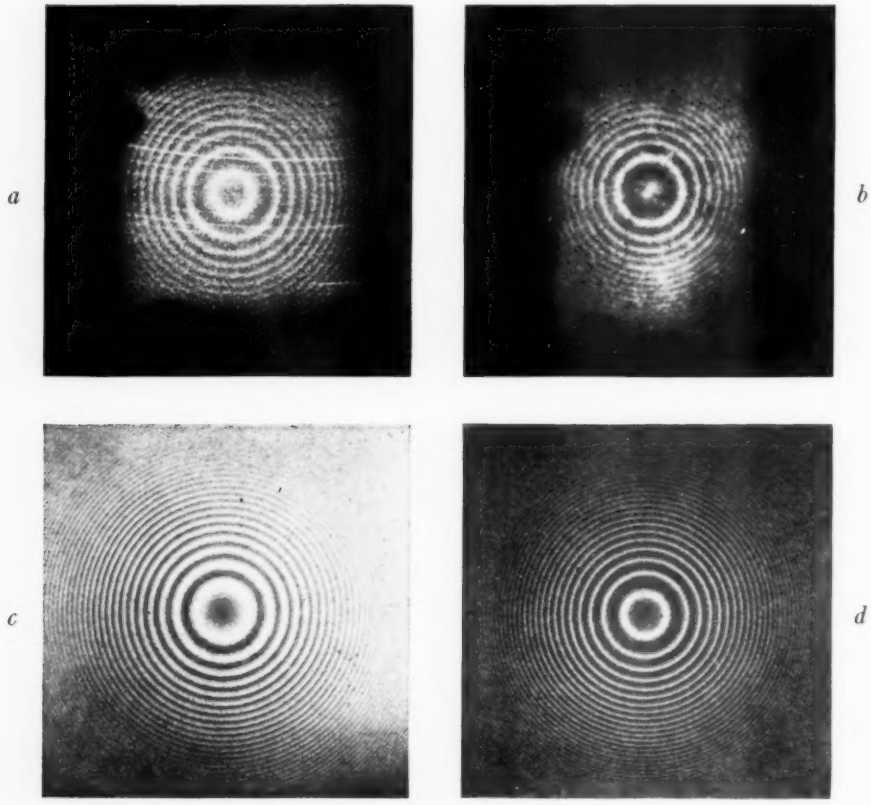
It has been pointed out[1] that for interferometer plates the scale in angstroms per millimeter at any point in the ring system is given by

$$\frac{d\lambda}{dr} = \frac{\lambda}{F^2} r$$

where F is the equivalent focal length of the lens which forms the image of the rings, and r is the radius to the point. For the present series of photographs, F has the value 76.8 mm, so that the coefficient of r is approximately 0.95. From this it is found that the

[1] *Mt. Wilson Contr.*, No. 137; *Astrophysical Journal*, **46**, 236, 1917.

PLATE XVII



INTERFERENCE RINGS FROM GREEN AURORAL LINE AND FROM ARTIFICIAL SOURCES

(*a*) Aurora. July 26–27, 1922, glass plate etalon, order of interference 15500. Exposure $6^h\ 50^m$, $f/1.9$.

(*b*) Aurora. October 25–26, 1922, air etalon, order of interference 8455. Exposure 10^h, $f/3$.

(*c*) λ 5461 of mercury, glass plate etalon, order of interference 15800.

(*d*) λ 5852 of neon, air etalon, order of interference 8058.

actual dispersion employed in these observations on the green auroral line varied from 8.5 A per mm for the largest ring used with the 1-mm etalon to 0.57 A per mm for the smallest one measured with the 8.7-mm etalon. The linear diameters of the interference rings were measured with a precision screw under a microscope fitted with a binocular eyepiece, at a magnification of about tenfold. On the best plates of the aurora as many as ten rings were measured, while on others only five were used. For the comparison plates, measurements were ordinarily made on from six to ten rings. The constant, $K\lambda$,[1] was determined from the comparison plates on account of the greater ease with which these plates could be measured, and the value of K to be used for the auroral rings was then derived from the ratio of the wave-lengths. The advantage of measuring several rings is especially great in the case of such photographs as those under discussion, for it largely eliminates the errors due to accidental distribution of silver grains in the emulsion and to slight imperfections in the film. Each ring furnishes an independent value of the fractional order, and thus of the wave-length, and, when reduced by the method indicated, all the results obtained from a given photograph are of practically the same reliability, out to a considerable distance from the center, depending upon the actual scale of the plates. No other measurements are required than those on the diameters themselves.

In Plate XVII are shown reproductions of the auroral photographs made on July 26–27, *a*, and on October 25–26, *b*, together with comparison photographs from mercury, *c*, and neon lines, *d*. The photographs of July 26–27 were made with a plane-parallel glass plate giving orders of interference of about 15,500 for the auroral line, and 15,800 for the green mercury line. The exposure time for the aurora was 6 hours, 50 minutes with the camera working at $f/1.9$. The plates of October 25–26 were made with an air etalon with orders of interference 8455 for the auroral line and 8058 for the yellow neon line (*d*). The exposure was 10 hours at $f/3$ for the aurora. The sky was partly cloudy during the last third of the exposure. A uniform enlargement of 4 diameters has been used in preparing these illustrations.

[1] *Mt. Wilson Contr.*, No. 202; *Astrophysical Journal*, **53**, 44, 1921.

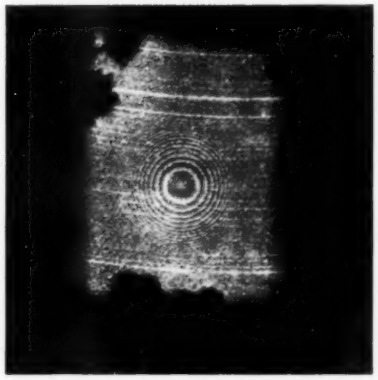
Plate XVIII shows at *a* the auroral exposure made on October 18–19 with an air etalon giving an order of interference of 31,000. At *b* and *c* are shown neon and mercury comparison plates used for determining the wave-length. At *d* is shown the auroral plate of November 17–18, made in Pasadena with a 1-mm etalon. The observation made on the aurora with the highest order of interference is not suited to reproduction, on account of the very small scale of the original.

The three primary data required concerning a spectral line are its wave-length, intensity, and width or character, whether single or complex, sharp or diffuse. The adaptability of the Fabry-Perot etalon to studies of all three kinds is well illustrated here. No other instrument offers such advantages in the problem at hand for precise measurements of wave-length and for examining the line width, while its usefulness for intensity observations is hardly secondary to that of a prism. It was long ago pointed out by Fabry and Perot[1] that the intensity of the maxima of the interference rings would be equal to that of the incident light if there were no absorption in the metallic films. If high resolving power is required, the reflecting power must be considerable, and this entails increased absorption, but for intensity observations the films may be made exceedingly thin, so that the loss by absorption is greatly reduced. The use of gold in place of silver for work in the green is a further help. Although measurements upon gold are not available it seems probable that an etalon employing this metal may be used for intensity observations with an efficiency of at least 60 per cent. The efficiency of a glass prism with its collimating lens cannot greatly exceed 75 per cent. This difference in favor of the prism is to a considerable extent offset by the fact that for weak images it is much easier to judge the blackening upon the interference pattern of the etalon than upon a single, narrow, and generally short image of the slit which is given by the prism.

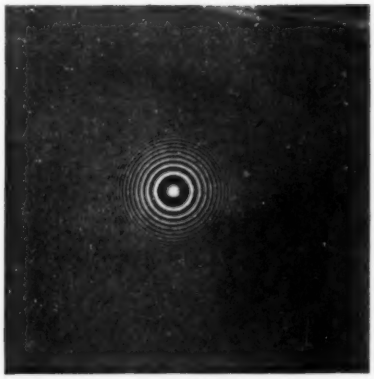
The value of the wave-length obtained here is not sufficiently different from that of Slipher to provide any obvious new possibilities for identifying the line, although it may prove an important aid in the solution of the difficulty. In this connection, it is interesting to

[1] *Annales de Chimie et de Physique* (7th series), **12**, 462, 1897.

PLATE XVIII



a



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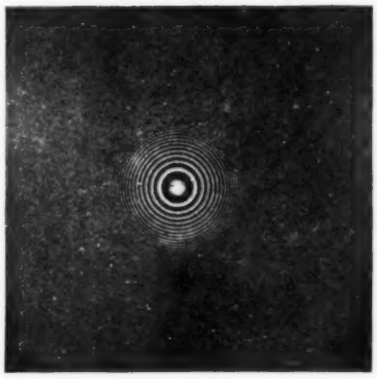
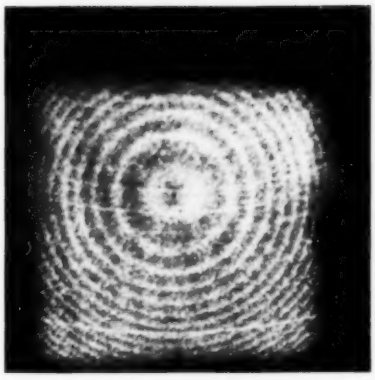


c



d

INTERFERENCE RINGS FROM GREEN AURORAL LINE AND FROM ARTIFICIAL SOURCES

(*a*) Aurora. October 18–19, 1922, air etalon, order of interference 31000. Exposure $9^h\ 30^m$, $f/3$.

(*b*) Plate for wave-length comparison; neon λ 5852, order of interference 29500.

(*c*) Plate for wave-length comparison; mercury λ 5461, order of interference 31600.

(*d*) Aurora. November 17–18, 1922, 1 mm air etalon, order of interference 3600. Exposure, made in Pasadena, 10^h, $f/3$.

recall the work of Fabry and Buisson[1] upon the relation between width of spectral line, atomic weight, and absolute temperature for a radiating gas. In their interesting paper they give results of laboratory tests of the theory that under suitable conditions of excitation the width is dependent solely upon the Doppler effect due to motion of the radiating centers in accordance with the kinetic theory of gases. The equation stated by them is

$$\Delta = 0.82 \times 10^{-6} \lambda \sqrt{\frac{T}{M}}$$

where Δ = width of line of wave-length λ; T = absolute temperature; M = atomic weight. Measurements are given for hydrogen, helium, neon, and krypton. In the case of hydrogen, there may be some question regarding the conclusion drawn, as Saha[2] has pointed out, but for the other three elements the observations appear to furnish satisfactory confirmation of the theory.

An application of this theory to the aurora, though not conclusive, may be of interest. If the absolute temperature of the earth's atmosphere at the height of the aurora is taken at 218 °C., as indicated by Humphreys,[3] and the width of the green auroral line is put equal to 0.035 A, the equation above gives for M the value 3.8, while if the width is subsequently found to be less than 0.035 A, the value of M will be increased accordingly. It is evident, then, that if hydrogen is postulated as the origin of the auroral line, the absolute temperature cannot much exceed one-fourth that of the isothermal layer. Helium, on the other hand, would correspond well with the probable temperature; but, if a heavier element is involved, the line-width must be expected to be less than the upper limit observed, since the absolute temperature can certainly not be much higher than 218°. It is possible that further studies of the line-width and an examination of the spectra, under extreme conditions of excitation, of the gases which are likely to be responsible may result in the discovery of a line corresponding to the new position found for the auroral line.

MOUNT WILSON OBSERVATORY
December 1922

[1] *Op. cit.*, **2**, 442, 1912.

[2] *Philosophical Magazine*, **40**, 159, 1920.

[3] *Physics of the Air*, p. 70

POLE-EFFECTS, PRESSURE-SHIFTS, AND MEASUREMENTS OF WAVE-LENGTHS IN THE SPECTRUM OF MANGANESE

By GEORGE SPENCER MONK

ABSTRACT

Interference measurements in the arc spectrum of manganese between λ 4470 *and* λ 6021.—Pole-effects, pressure-shifts, and wave-lengths for the stronger lines of manganese between λ 4470 and λ 6021 have been measured. A carbon arc with the lower, positive, pole bored out and filled with manganese dioxide was used as a source. (1) *Pressure-shift:* The interference method used does not seem to be suitable for observations on pressure-shift in manganese, on account of the intrinsic wideness of many lines, and their widening under pressure. The evidence obtained indicates a correspondence between unsymmetrical broadening of the lines under pressure, and pole-effect. (2) *Pole-effect:* Little or no pole-effect is observed with a carbon arc whose lower pole is filled with manganese chloride. There are objections, however, to its use. A moderate effect is observed at the positive pole of a carbon arc filled with manganese dioxide, and a larger effect with an arc between metallic poles. (3) *Wave-lengths:* Only about 1 millimeter of the center of a 12 mm to 15 mm, 5-ampere arc on a 220-volt circuit was used, and it is believed that pole-effect is entirely absent from the measurements. The wave-lengths of about forty lines have been measured with an accuracy estimated at 0.002 A. Lines of neon and lines of iron of group *a* were used as standards, the strongest component of Hg 5461 serving as an auxiliary standard.

On account of the prevalence of manganese as an impurity in iron, and the consequent appearance in the spectrum of iron of the stronger manganese lines, the spectrum of manganese is of interest as a possible source of standards of wave-length. Also, a knowledge of the correct wave-lengths of many manganese lines is necessary for comparison with the corresponding wave-lengths in the sun. This paper contains the results of interferometer measures of the wave-lengths of a number of lines in the region λ 4470 to λ 6021, together with an examination of the stronger lines in that region for pole-effect, and their behavior under moderate differences of pressure.

Wave-lengths of manganese lines have been published by a number of observers. Chief among these are those due to Exner and Haschek,[1] Kilby,[2] and Fuchs,[3] while some of the stronger lines

[1] *Die Spektren der Elemente*, Leipzig, 1912.

[2] *Astrophysical Journal*, **30**, 243, 1909.

[3] *Zeit. für Wiss. Phot.*, **14**, 239, 1915.

have been measured as impurities in the arc spectrum of nickel by Hamm,[1] and of iron by Burns,[2] who used the interference method.

THEORY AND APPARATUS

From elementary theory, it is easily deduced that the fractional order of interference at the center of the ring system formed by the passage of light through a parallel-plate interferometer is expressed by

$$E+(n-1)=\frac{D_{n-1}^2}{D_n^2-D_{n-1}^2}(1+D_n^2/8F^2m^2)\,, \qquad (1)$$

where E is the fractional part of the whole order of interference at the center of the ring system, D is the diameter of a ring, F is the focal length of the lens or mirror projecting the fringes on the slit of a spectrograph, m is the magnification of the spectrograph, and n is an integer referring to the number of the ring counting from the center outward. This equation holds only when the magnification of the spectrograph is constant for all wave-lengths. For the apparatus used in this investigation, F is 500 mm, the maximum D is 25 mm, and m is 1.07. Hence, neglecting the second term in the bracketed factor on the right-hand side of equation (1) introduces a negligible error of one part in about thirty-five hundred for the largest fringe measured. Also, it can easily be shown that the difference between the squares of the diameters of contiguous fringes for a given wave-length is a constant. Hence, equation (1) can be put into the simpler form

$$E+(\text{integer})=\frac{D^2}{D_k^2-D_{k-1}^2} \qquad (2)$$

in which the denominator is the average of all the differences of diameters squared for a given line. Thus, by the use of equation (2), as many determinations of E, the fractional order of interference at the center of the ring system for a given line, may be made as the number of fringe diameters measured. In practice, this number was limited to eight or nine. Equation (2) is due to St. John and Babcock, and its use has been explained by them.

[1] *Zeit. für Wiss. Phot.*, **13**, 105, 1913. [2] *Ibid.*, **12**, 207, 1913.

The conditions in the foregoing theory are present in the apparatus used. Light from a source L (Fig. 1) is focused on the aperture of a parallel-plate interferometer by a concave mirror C_1, of 17 cm diameter and 60 cm focal length. By means of a plane mirror M, the light is then reflected to concave mirror C_2, which is set at a distance from the slit S equal to its focal length. Thus the fringes produced at the interferometer are in focus on the slit. For most of the plates, C_2 is of 50 cm focal length, although some were taken with mirrors of 60 cm and 25 cm focal length. Auxiliary

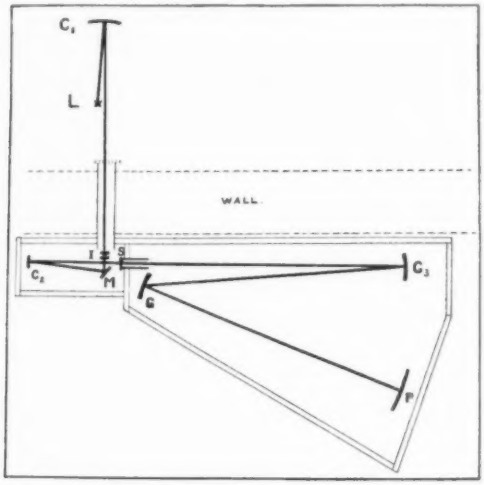


FIG. 1

dispersion of the fringes for different wave-lengths is obtained by the use of a concave grating G, on which the incident rays are made parallel by the use of the concave mirror C_3, of approximately 6 meters focal length. This form of the apparatus is due to Pfund. The grating, ruled by Professor Michelson, is of good quality and has a ruled surface 2 inches by 5.5 inches, with 15,000 lines to the inch. Its focal length is 21 feet. A negligible amount of astigmatism exists in the spectrograph owing to the small angle, about two degrees, between the incident and reflected beams at C_3. The grating and plate-holder are mounted at the ends of a rigid steel bar, which is movable about a vertical axis passing through the

center of the grating. The photographic plate is 10 inches long by 2 inches wide, bent so that the fringes are in focus over its entire length. The plate-holder is so mounted in guides that it can be moved a small distance horizontally in a direction at right-angles to the beam of light by means of a push-screw, thus permitting exposures, with the source under different conditions, to be photographed so that the spectrum lines are side by side on the same plate. The housing of the auxiliary spectrograph is large enough to permit exposures in the yellow of the second order.

The interferometer is equipped with a number of etalons ranging in thickness from 2.5 mm to 40 mm. The etalons are brass rings with three invar studs exactly equal in length to within a fraction of a light-wave. During this investigation, a pair of quartz mirrors was used, accurately flat to a fraction of a fringe, and sputtered to an appropriate thickness of silver by cathodic deposition. The aperture of the interferometer, upon which the light of the source is focused, is placed between the source and the silvered surfaces, and as near the latter as possible.

The entire instrument, inclosed in a tight, double-walled box, is located in a constant-temperature room in the basement of Ryerson Laboratory. The source of light and the concave mirror C_1 are in an adjoining room, the light being projected on the interferometer through a hole in the wall, over one end of which is a plate-glass cover. With these precautions, the interior of the interferometer box and spectrograph is free from temperature effects sufficient to cause displacements of the fringes. In order to obtain a check on instrumental changes of any sort during exposures, a short exposure to a Cooper-Hewitt lamp was made on each plate before and after each of the principal exposures. In addition, the order in which the exposures to manganese arc and standards were made was varied, to prevent systematic errors due to possible changes. Some few plates were taken with simultaneous exposures of manganese and standard source, but it was found that successive exposures with the Hg spectrum as a check gave satisfactory results. It might be mentioned in this connection that the character of the lines examined made the highest degree of accuracy of measurement impossible. Where the character of the inter-

ference fringes and the nature of the source permit the greatest accuracy, simultaneous exposures are to be recommended.

In all, about fifty plates of the manganese spectrum were taken and measured. The source was a vertical carbon arc, mounted so that quick and accurate adjustments could be made for focus and setting of the image of the source on the interferometer aperture. In the lower, positive, electrode, about 12 mm in diameter, was drilled a 3 mm hole, packed with MnO_2. The upper, negative, electrode was a solid carbon, 9 mm in diameter. Attempts were made to use a metallic manganese arc, in the hope that the extremely steady conditions of the Pfund iron arc could be obtained, but metallic manganese spluttered so badly that it proved unmanageable. Some exposures were made with a carbon arc as described, the lower pole being charged with manganese chloride instead of manganese dioxide. This arc required too frequent charging, and the location of the center of the arc was an uncertain matter. The manganese dioxide formed a bead almost equal in steadiness and longevity to the bead of the Pfund iron arc. The current was maintained at 5 amperes on a 220-volt circuit, and the distance between the electrodes was from 12 to 15 mm.

For the red region, Wratten Panchromatic plates were used. Cramer old-style X-ray plates, having a pronounced maximum of sensitiveness to the green, were used for wave-lengths below λ 5600. Exposure times ranged from about four minutes for the brightest component of Hg λ 5461 to an hour for some of the faint lines of manganese.

The etalons used were 5 mm and 7.5 mm. The choice of these distances rather than distances with a greater difference between them was necessitated by the character of the lines measured. Most of them were intrinsically very wide. The principal effect of the small range is an increase of the uncertainty of the phase-change correction. A comparison of the measures with those of other observers indicates that no systematic error has been thus introduced.

The image of the source in focus on the interferometer was about 5.5 times the size of the source. This gave an image of the arc between 70 and 80 mm in length. The aperture of the interferometer was rectangular in shape, and 6 mm high by 13 mm wide.

Thus, the central portion of the arc selected was well within the dimensions prescribed by the Committee on Standard Wave-Lengths. The interferometer was so placed that an image of its aperture coincided with the ruled area of the grating and was in sharp focus thereon. It is believed that on account of the large magnification of the arc, and the careful selection of its center, the wave-lengths determined are entirely free from pole-effect.

MEASUREMENT AND COMPUTATION

The plates were measured on a Gaertner comparator, readings being made to hundredths of a millimeter. Two or three settings were made and averaged for each fringe. In some exceptional cases, such as in preliminary determinations of the phase-change correction where the etalon used was so small that a mirror of comparatively short focal length was used to project the fringes on the slit, the readings were made to thousandths of a millimeter. From the measured diameters of the fringes, the fractional orders of interference were computed to the third decimal place according to formula (2). The integral part of the order of interference was computed on a calculating machine by a quick and easy method similar to that described by Meggers. The complete order of interference thus obtained, multiplied by the wave-length of the standard line, gave the double distance between the interferometer plates. This value, divided by the orders of interference for the other lines on the plate, gave the wave-lengths of those lines to the fourth decimal place.

As standards, in the red, lines of neon were used, in particular λ 5944.834; in the green, the Fe line λ 5497.522 was used as a standard, although many of the plates were measured in terms of Hg λ 5461, this line being measured in terms of the Fe and neon lines for both etalons. The adopted values of the wave-length of the bright component of Hg λ 5461 were: for the 5 mm etalon, λ 5460.7443; for the 7.5 mm etalon, λ 5460.7434. The difference is due to a greater nearness of a fainter component for one of the etalons. Both of these values are slightly larger than the value λ 5460.7424 obtained by Fabry and Buisson with a 5 mm etalon. A number of plates were taken with neon, mercury, and iron, giving a check on the phase-change corrections used. A curve

plotted for the corrections thus derived agrees well with that obtained from the manganese measures. In compiling the final table of wave-lengths, the formula for the phase-change correction derived by Meggers was used. Table I shows the application of the formula to three lines and a comparison with the values obtained by calculating the phase-change correction for the region on the basis of the neon lines, assuming the Bureau of Standards values as correct.

TABLE I

REDUCTION FOR PHASE-CHANGE

$$\Delta_1=\frac{e_2 c}{e_2-e_1}; \qquad \Delta_2=\frac{e_1 c}{e_2-e_1}; \qquad e_1=5.0 \text{ mm}; \qquad e_2=7.5 \text{ mm}$$

c=difference between uncorrected wave-lengths of a line for two etalons

LINE	UNCORRECTED		c	CORRECTED	BY NEON LINES	ADOPTED
	7.5 mm	5.0 mm				
6013.....	.4964	.4978	−0.0014	.4936	.4946	.494
6016.....	.6459	.6474	−0.0015	.6432	.6440	.644
6021.....	.8015	.8028	−0.0013	.7987	.7995	.799

PRESSURE-SHIFTS

Table II gives the results of measures on a number of lines photographed with the source subject to pressures ranging from 5 cm to 160 cm of mercury. A plate taken with a metal electrode showed fringes so widened for moderate pressures that measures were impossible. It was not considered advisable to make a more comprehensive study of pressure-shifts in manganese at this time in view of the small extent of the region investigated.

TABLE II

Line	Displacement per Atmosphere High-Low	Line*	Displacement per Atmosphere High-Low	Line	Displacement per Atmosphere High-Low
4709.....	0.004	4783.....	0.004	5470.....	0.007
4727.....	0.004	4823.....	0.006	5481.....	0.008
4739.....	0.002	5341.....	0.004	5516.....	0.003
4754.....	0.006	5377.....	0.008	5537.....	0.008
4761*	0.014	5394.....	0.006	6013.....	0.009
4762*	0.006	5399.....	0.009	6016.....	0.011
4765*	0.009	5420.....	0.010	6021.....	0.013
4766*	0.006	5432.....	0.003		

* Close pairs, measures doubtful.

It is important to note a fact which does not appear from Table II. The displacements for λ 4754, λ 4783, and λ 4823 are from settings on what appears to be the center of the most dense portion of the fringes. Actually, in pressures higher than atmospheric, these fringes were drawn out so far toward the center of the ring system for each of the lines that it was doubtful where the center of the line really lay. In other words, these three lines were unsymmetrically widened toward the red, a condition frequently noted in certain groups of lines photographed under high pressures, and strikingly illustrated in the plates of Gale and Adams[1], taken with the source under a pressure of 8 atmospheres. The displacements indicated in Table II for λ 6013, λ 6016, and λ 6021 are based on observations at atmospheric pressure or less, no measures being made on plates taken at higher pressures.

POLE-EFFECT

The pole-effect for manganese has been investigated by Brendel-Wirminghaus[2] for wave-lengths shorter than λ 5600. This investigator used a carbon arc, one pole drilled out and filled with manganese chloride. She concluded from her results that the spectrum of manganese showed no pole-effect. The present writer made a number of exposures with this form of arc, and observed little or no pole-effect at the negative, carbon, pole. At the positive pole, which was drilled out and filled with manganese chloride, the results were inconclusive, a few plates showing an appreciable effect, and many, none at all. An objection to the use of manganese chloride is in the uncertain manner in which the arc burns. Most of the time, since the chloride is used up quickly, the arc issues from the positive pole at a point between 2 and 5 mm below the end of the electrode, rendering observations for pole-effect useless.

With a metallic arc, the values shown in Table III have been obtained, using the third order of the 10-inch Michelson grating in the 30-foot Littrow spectrograph. A comparison with the results of St. John and Babcock is given in Table III. Several exposures

[1] *Mt. Wilson Contr.*, No. 58; *Astrophysical Journal*, **35**, 10, 1912.

[2] *Zeit. für Wiss. Phot.*, **20**, 229, 1921.

using the interference method with a carbon arc, in which the lower, positive, pole was drilled out and packed with manganese dioxide, gave the results in Table IV.

TABLE III

Line	Monk	St. John and Babcock
	Neg. Pole-Center	Neg. Pole-Center
6013	0.018	0.019
6016	0.017	0.020
6021	0.019	0.018

TABLE IV

Line	No. Plates	Positive Pole-Center	Line	No. Plates	Positive Pole-Center
4472	1	.003	5255	2	.003
4490	1	.003	5341	2	.003
4498	1	.003	5377	2	.008
4502	1	.004	5394	3	.003
4709	3	.002	5420	3	.005
4727	3	.004	5470	3	.003
4739	3	.004	5481	3	.006
4754	2	.008	5505	2	.005
4783	2	.009	5516	4	.005
4823	2	.007	6013	4	.007
5150	2	.005	6016	4	.008
5196	2	.005	6021	4	.012

λ 4754, λ 4783, λ 4823, which were mentioned in connection with the discussion of pressure-shifts, as having an abnormal widening toward the red under pressure, are here shown to have

(*a*) (*b*) (*c*)

FIG. 2

larger pole-effects than other lines in the same region. It is also of interest, in connection with pole-effect, to notice the difference in appearance of the three types of arc used. These are illustrated

in Figure 2: (*a*) is almost uniform in intensity from pole to pole, the arc at the positive pole seeming to spring from the crater in which the manganese chloride has sunk; (*b*) has a marked condensation of the discharge at the point on the manganese oxide bead from which the arc springs, and is brighter there; (*c*) has this same feature at the positive pole, and an excessively great intensity at the negative pole.

WAVE-LENGTHS

The adopted wave-lengths for the lines measured are shown in Table V, compared with the results of other observers. A difference of 0.1 A in the case of λ 4739 is undoubtedly due to an error or misprint in the lists of other observers, since no manipulation of the whole number part of the order of interference will give an approximation to their results. The value for λ 5377 is in some doubt owing to the presence of a faint line close on the violet side. While the probable errors in column 8, based on the collected measures from each plate, are as low as 0.001 A, this does not represent the consistency of the measures of the individual fringes for a single exposure. These were not so good, especially for the very wide lines. Consequently, it is believed that claim cannot be made to greater accuracy than 0.002 A.

The usefulness of manganese lines in the region studied as possible secondary standards is in general impaired by two considerations. Many of the lines are eliminated on account of their intrinsic width. Most of those which are designated by Janicki as sharp, simple lines are as a rule most affected by pressure-shift and pole-effect. Among these latter are λλ 4754, 4783, 4823, 6013, 6016, 6021, which on account of their intensity, appear most frequently in the arc spectrum of iron. With exposures carefully made on the center of the arc, they should be useful as standards of measurement.

No lines of any prominence were measured in the region between λ 5537–λ 6013. While there are several lines of moderate intensity in the neighborhood of λ 5800, none of the plates taken showed them with sufficient density for measurement. Three other lines, λ 4709, λ 4727, and λ 4739, all sharp, and having the advantage of comparative isolation in the spectrum, gave promise of usefulness

as standards. The values obtained for these lines are however so different from the values obtained by other observers that

TABLE V

Line	Rowland	I. A. Units					Prob. Error	No. Plates
	Exner & Haschek	Kilby	Fuchs	Burns	Hamm	Monk		
4470	.33	.138	.142	.141	.147	.140	.0006	7
4472	.98	.800	.793	.....	.....	.796	.0004	7
4490	.27	.071	.078	.....	.....	.081	.0002	6
4498	.09	.898	.897	.903	.....	.900	.0004	6
4502	.40	.218	.223	.219	.228	.222	.0003	7
4605	.52	.378	.367	.....	.....	.365	.0005	6
4626	.69	.552	.546	.....	.....	.542	.0004	6
4671	.89	.694	.688	.....	.....	.684	.0008	2
4701	.31	.150	.158	.....	.....	.154	.0008	3
4709	.89	.708	.704	.....	.....	.713	.0002	11
4727	.70	.476	.462	.464	.....	.479	.0002	11
4739	.30	.004	.001	.....	.....	.106	.0002	11
4754	.24	.046	.048	.....	.....	.041	.0004	6
4761	.73	.521	.527	.524	.528	.526	.0002	6
4762	.60	.375	.375	.373	.374	.375	.....	2
4765	.08	.852	.856	.863	.861	.861	.0003	6
4766	.63	.414	.426	.422	.421	.429	.0004	5
4783	.62	.454	.432	.434	.....	.428	.0006	6
4823	.71	.521	.522	.....	.....	.516	.0006	6
5117	.10	.944	.935	.....	.....	.940	.....	1
5150	.10	.937	.....	.....	.....	.930	.....	2
5255	.48	.330	.330	.....	.....	.320	.0004	4
5341	.25	.068	.070	.....	.....	.066	.0012	7
5377	.86	.623	.634	.....	.....	.619	.0007	11
5394	.89	.679	.677	.....	.....	.676	.0006	15
5399	.70	.494	.506	.....	.....	.487	.0006	9
5407	.67	.429	.432	.....	.....	.428	.0009	4
5413	.90	.690	.696	.....	.....	.683	.0010	5
5420	.61	.371	.368	.....	.....	.361	.0007	6
5432	.75	.553	.555	.....	.....	.545	.0006	11
5457	.64	.....	.468	.....	.....	.463	.....	1
5470	.88	.644	.640	.....	.....	.642	.0005	8
5481	.61	.401	.395	.....	.....	.387	.0006	5
5505	.10	.874	.877	.....	.....	.874	.0006	9
5516	.00	.774	.773	.....	.....	.773	.0005	11
5537	.99	.753	.749	.....	.....	.759	.0007	9
5551	.20	.991	.987	.....	.....	.984	.....	1
5567	.02	.....	.764	.....	.....	.761	.....	1
6013	.74	.480	.484	.....	.....	.494	.0006	12
6016	.90	.631	.636	.....	.....	.644	.0005	13
6021	.05	.794	.787	.....	.....	.799	.0008	14

further measurement is necessary. It is planned to continue this investigation in the arc spectrum of manganese, using a cadmium

tube as a standard source, for such lines as these, for fainter lines, and for other parts of the spectrum.

In conclusion, the writer wishes to acknowledge the help and guidance of Dr. H. G. Gale, under whose direction this work was begun and carried on, and whose liberality in time and equipment made the results possible. Thanks are also given to Messrs. St. John and Babcock, who furnished details of their own apparatus, to Dr. Elias Klein for taking and measuring several plates, and to Mrs. Monk for aid in measuring and computing.

RYERSON LABORATORY
April, 1923

A NEW METHOD FOR THE MEASUREMENT OF THE DEPTH OF THE CHROMOSPHERE[1]

By PHILIP FOX

ABSTRACT

Depth of the chromosphere.—(1) *Prism micrometer method of measurement* involves reflecting light from each of two diametrically opposite edges of the solar image on to the slit of a spectroscope by means of prisms whose distance apart can be varied by screw adjustments. The apparatus is shielded from the sun's heat by means of a plate equal in size to the solar image. The double thickness of the reversing layer is determined from the difference between the minimum and maximum separations of the prisms for which light from each shows the chromosphere lines. (2) *Preliminary observations* at Yerkes Observatory, under unfavorable conditions of seeing, indicate that for a clear sky the depth would be over 10″ for Hα and over 8″ for D_3.

In 1909 Hale and Adams published a paper on the "Photography of the Flash Spectrum without an Eclipse."[2] In this paper they described the apparatus and method of operation for holding the limb of the sun exactly tangential to the slit of the spectrograph. The success of the observations depends on holding the bright lines of the chromosphere continually in view during the exposures. This is a matter of considerable delicacy, for, if the photosphere falls on the slit, its brilliant spectrum will blot out the fainter chromospheric lines, and if the slit is too far from the limb the lines of the lower-lying vapors do not appear.

The success of their observations suggested that if such an apparatus could be adapted for simultaneous visual observations of the "flash" spectrum from points on the limb diametrically opposite we would have a very delicate method of measuring the depth of the reversing layer and possibly also the diameter of the sun.

As the only observational evidence on the thickness of strata of vapors of the reversing layer heretofore available comes from the length of cusps shown by objective-prism spectra of the chromosphere taken at times of solar eclipses,[3] it was thought advisable to test this new suggestion with a view to supplementing these results.[4]

[1] This method was first suggested by the writer at the Washington meeting of the American Section of the International Astronomical Union on April 4, 1922.

[2] *Astrophysical Journal*, **33**, 222–230, 1909.

[3] Frost, *Astrophysical Journal*, **12**, 321, 1900; S. A. Mitchell, *ibid.*, **15**, 119, 1902.

[4] Since writing this paragraph my attention has been directed to St. John's observations of the depth of the reversing layer in which he used photographs of spectra made with radial slit. *Astrophysical Journal*, **32**, 64, 1910.

Various observers in studying spectroscopically the problem of the rotation of the sun have used a device[1] which can be easily adapted to the present needs. Certain features of the apparatus used by the investigators of the flash spectrum and of the solar rotation were combined in a device, which may be called a prism micrometer, shown in Figure 1. This was made up by Mr. Küng in the shops of the Physical Laboratory of Northwestern University. The set of prisms was loaned to Dearborn Observatory through the courtesy of Professor H. D. Curtis, of Allegheny Observatory.

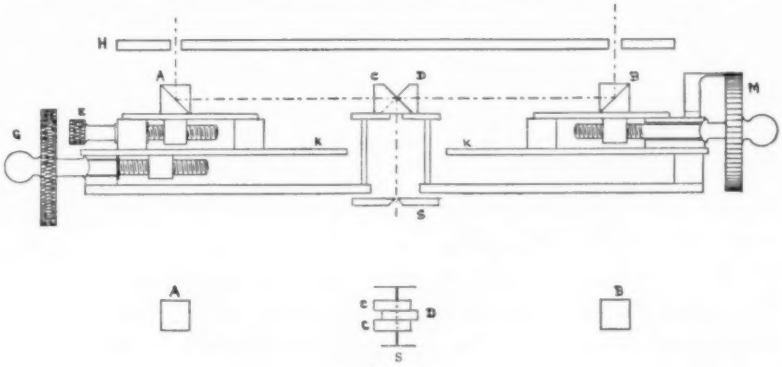


FIG. 1

Prisms *A* and *B*, Figure 1, reflect light from the limb of the sun respectively to prisms *C* and *D*, which are mounted in fixed position over the slit of a spectroscope *S*. Prisms *A* and *B* are set at approximately equal distances from the slit and are separated by a distance about equal to the diameter of the solar image; the adjustments are made by means of screw *E* and the micrometer screw *M*. After the approximate adjustment is made, the screw *E* is not again touched. The carriage *K*, on which the two prisms *A* and *B* are mounted, can be moved to follow the image by means of the screw *G*. Any considerable movement to this screw will lengthen the light path for one limb and shorten that from the other. If the driving clock is well rated, this maladjustment of focus should not be serious. The plate *H* is to shield the apparatus from the sun's heat.

[1] W. S. Adams, Carnegie Institution of Washington, *Publication No. 138*, 3, 1911; Schlesinger, *Allegheny Observatory Publications*, 3, 103, 1914.

In determining the thickness of the strata of vapors of the reversing layer, the process of measurement is to bring the sun central on the shield *H;* by means of the screw *G* the carriage is moved until the chromospheric spectrum from *A* is visible in the spectroscope; holding this with the screw *G*, prism *B* is moved by the micrometer screw *M* until the same spectrum from *B* is also seen. The setting of *M* is read for the smallest distance between *A* and *B* where these spectra from both limbs are visible. Prisms *A* and *B* are then separated by *M*, always adjusting with *G*, until the maximum diameter for simultaneous visibility of the spectrum from both limbs is reached. At this point *M* is again read. The difference of these two readings of *M*, with perhaps some correction dependent on slit width, will give the double depth of the reversing layer for the particular line under observation.

Professor Frost offered the facilities of the Yerkes Observatory for carrying out this investigation. Inasmuch as the advantages of the large solar image given by the 40-inch refractor are obvious, his offer was accepted and the observing has been carried on there. The prism micrometer was mounted before the slit of the auto-collimating stellar spectroscope of that Observatory. This spectroscope has a lens of 30 inches focal length. The dispersion piece is a Michelson grating, which was used in the first order. The apparatus was assembled in the early days of September. The adjustments offered some difficulty and even when these were completed it was found that the operation of the instrument was quite inconvenient. The observations were made during two visits to the Yerkes Observatory, in the middle and end of September. On neither occasion was the sky favorable; observations were made in intervals between clouds or through thin clouds or haze. It was found possible, however, to measure the depth of the reversing layer for the hydrogen line Hα and for the helium line D_3. The results are collected in Table I.

All of the measures were made in the north and south direction. The slit of the spectrograph was set at widths between 0.05 mm and 0.08 mm. The aperture of the telescope was reduced to about 20 inches.

It is seen that the results have a fairly large range. The smaller values, in general, were obtained when the sky was thick and the

sky spectrum, therefore, very bright. Under such circumstances, the bright line of the reversing layer is lost at no very great distance from the limb, on account of the diminished contrast. Under uniform conditions, the results seem to be fairly accordant. It is a safe estimate that under good atmospheric conditions, high transparency and steadiness, the Hα line could be observed consistently at depths of 10″ and D_3 fully 8″. The chromospheric

TABLE I

MEASURED THICKNESS OF THE REVERSING LAYER

1922	Line	Double Depth	Single Depth	Remarks
		R		
Sept. 15	Hα	4.1	8.″5	Through breaks in clouds. With Dr. Lee
		4.5	9.3	Through breaks in clouds
		4.6	9.5	Through breaks in clouds. With Dr. Lee
		4.2	8.7	Through breaks in clouds
		4.7	9.7	Through breaks in clouds
		4.5	9.3	Through breaks in clouds
Sept. 16		5.2	10.8	
Sept. 17		5.2	10.8	
		4.7	9.7	Clouds at end
Sept. 30		4.6	9.5	Hazy
		4.5	9.3	Hazy
		4.6	9.5	Hazy
Oct. 1		4.4	9.1	Very hazy
		4.7	9.7	With Professor Barnard
Sept. 15	D_3	2.8	5.8	Thin clouds
		2.5	5.2	Thin clouds
Sept. 16		4.1	8.5	Fairly transparent. Cloud intervals
		3.9	8.1	Fairly transparent. Cloud intervals
Sept. 30		2.5	5.2	Very thick, high cirrus clouds
Oct. 1		3.0	6.3	Thick
		3.2	6.6	Thick

reversals for certain other lines of the spectrum were also seen, but under the unfavorable conditions of the sky it seemed useless to try to use them for measurement. The results which are tabulated above must be considered as of preliminary character. It is hoped to extend the series of observations at a later date.

I wish to acknowledge with thanks the assistance given in this investigation by Professors Curtis, Frost, and Crew.

DEARBORN OBSERVATORY
October 9, 1922

DISPERSION OF LIGHT BY AN ELECTRON GAS

By LEIGH PAGE

ABSTRACT

Electro-magnetic theory of the dispersion of an electron gas is deduced. The phase velocity in such a gas is found to be a function of the wave-length slightly greater than the velocity of light in a vacuum. From Shapley's estimate of a difference between the velocities of blue and yellow light not greater than 0.1 meter per sec., an upper limit to the *number of electrons per cc in interplanetary space* can be computed. This number is $3(10)^{11}$ for electrons or $5(10)^{14}$ for protons. The expressions obtained for the electric and magnetic vectors are checked both by the law of conservation of momentum and that of energy.

From the agreement in phase of the blue and yellow light curves of distant stars Shapley[1] has drawn the conclusion that the velocity of blue light through space cannot differ from that of yellow light by as much as one-tenth of a meter per second. While it is not likely that any considerable amount of matter in the atomic or molecular state is present in interstellar space, it may well happen that certain regions are traversed by swarms of electrons or protons. Hence it seems of interest to investigate whether the laws of electromagnetism lead to dispersion of light passing through an electron gas. For the sake of simplicity the electron gas will be supposed to be of uniform density, and the light will be assumed to be plane polarized. In connection with the exponentials employed, the real part alone will be understood to be taken. The electromagnetic quantities are expressed in Heaviside-Lorentz rational units.

Consider a train of plane waves traveling in the X direction. Let the electric vector have the Y direction. Then, at any one point,

$$E_y = E_0 e^{-i\omega t}.$$

Suppose an electron to be present at this point. If v denotes the velocity of the electron in the Y direction due to the alternating electric field of the wave, the equation of motion of the electron is

$$N\frac{d^2v}{dt^2} - M\frac{dv}{dt} = -E_0 e^{-i\omega t},$$

[1] *Bulletin Harvard Coll. Observatory*, No. 763, Jan. 1922.

where

$$M \equiv \frac{m}{e}, \qquad N \equiv \frac{e}{6\pi c^3},$$

m and e designating the mass and charge respectively.

Hence, as $N\omega$ is small compared to M,

$$v = \frac{iE_0}{M\omega} e^{-i(\omega t+\delta)},$$

where

$$\delta \equiv \tan^{-1} \frac{N\omega}{M}.$$

If n denotes the number of electrons per unit volume, the density of charge ρ is equal to ne, and

$$\rho v = \frac{ineE_0}{M\omega} e^{-i(\omega t+\delta)}$$

$$= -\frac{ne}{M\omega^2} e^{-i\delta} \dot{\boldsymbol{E}}. \tag{1}$$

Now, the equations of the electromagnetic field are

$$\nabla \cdot \boldsymbol{E} = \rho, \tag{2}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c} \dot{\boldsymbol{H}}, \tag{3}$$

$$\nabla \cdot \boldsymbol{H} = 0, \tag{4}$$

$$\nabla \times \boldsymbol{H} = \frac{1}{c} (\dot{\boldsymbol{E}} + \rho \boldsymbol{v}). \tag{5}$$

Substituting in (5) the value of $\rho \boldsymbol{v}$ given by (1), this equation becomes

$$\nabla \times \boldsymbol{H} = \frac{1}{c} \left(1 - \frac{ne}{M\omega^2} e^{-i\delta}\right) \dot{\boldsymbol{E}}. \tag{5'}$$

Eliminating $\boldsymbol{H}$ by the usual method, taking note of the fact that ρ is constant and hence

$$\nabla \nabla \cdot \boldsymbol{E} = \nabla \rho = 0,$$

it is found that

$$\nabla \cdot \nabla \boldsymbol{E} - \frac{1}{c^2} \left(1 - \frac{ne}{M\omega^2} e^{-i\delta}\right) \ddot{\boldsymbol{E}} = 0,$$

and an exactly similar equation in $\boldsymbol{H}$ is obtained by eliminating $\boldsymbol{E}$. These are the equations of a wave traveling with wave slowness

$$S=\frac{1}{c}\left(1-\frac{ne}{M\omega^2}e^{-i\delta}\right)^{\frac{1}{2}}.$$

Put the complex wave slowness equal to $S'+iS''$, where S' and S'' are real. Then, provided $\frac{ne}{M\omega^2}$ is small compared to unity,

$$q=\frac{1}{S'}$$

$$=c\left(1+\frac{ne^2}{2m\omega^2}\right) \tag{6}$$

is the velocity of propagation, and

$$\omega S''=\frac{neN}{cM^2} \tag{7}$$

is the coefficient of absorption.

Equation (6) shows that the phase velocity is greater than the velocity of light *in vacuo* in much the same manner as in the case of a dielectric in the region of anomalous dispersion. Written in terms of wave-length this equation has the form

$$q=c\left(1+\frac{ne^2}{8\pi^2mc^2}\lambda^2\right).$$

If now, q_B represents the velocity of blue light, and q_Y that of yellow light,

$$q_Y-q_B=\frac{ne^2}{8\pi^2mc}(\lambda_Y^2-\lambda_B^2).$$

Taking 0.1 meter/sec. as the greatest allowable value of the difference of the two velocities, an upper limit to the number n of electrons per cubic centimeter may be calculated. This number is $3(10)^{11}$ per cubic centimeter for electrons, and $5(10)^{14}$ for protons, the latter having a smaller dispersive effect on account of their greater mass.

The electric and magnetic field strengths in the wave are

$$E_y = Ae^{-\omega S'' x} e^{i\omega(S'x-t)},$$

and

$$H_z = Be^{-\omega S'' x} e^{i[\omega(S'x-t)+\epsilon]},$$

where

$$B \equiv Ac\sqrt{S'^2+S''^2},$$

$$\epsilon \equiv \tan^{-1}\frac{S''}{S'}.$$

These expressions may be checked by both the law of conservation of momentum and that of energy. Consider a layer of the electron gas of unit cross section at right angles to the direction of propagation of the light and of thickness δx. The loss in momentum of the radiation passing through this layer in a unit time is

$$\tfrac{1}{2}AB - \tfrac{1}{2}AB(1-2\omega S''\delta x)$$

$$\doteq \tfrac{1}{2}A^2\frac{\rho}{c}\frac{N}{M^2}\delta x.$$

Dividing this by the number $n\delta x$ of electrons in the layer, the loss in momentum per unit time per electron is

$$\tfrac{1}{2}A^2\frac{e}{c}\frac{N}{M^2}.$$

But this is just the radiation pressure on an electron.[1]

The energy lost by the waves which pass through this layer in a unit time is

$$\frac{c}{2}AB - \frac{c}{2}AB(1-2\omega S''\delta x)$$

$$\doteq \tfrac{1}{2}A^2\rho\frac{N}{M^2}\delta x$$

or, dividing by the number of electrons, the energy lost to each electron is

$$\tfrac{1}{2}A^2 e\frac{N}{M^2}.$$

[1] Leigh Page, *Astrophysical Journal*, **52**, 67, 1920.

Now, the energy radiated away by each electron in the course of its oscillations under the influence of the electric field in the wave is

$$\frac{e^2}{6\pi c^3}\left(\frac{dv}{dt}\right)^2$$

$$=\tfrac{1}{2}A^2e\frac{N}{M^2}$$

per unit time.

The energy transferred to the electrons through the agency of light pressure is of a smaller order of magnitude and does not appear on account of the approximations made.

SLOANE PHYSICS LABORATORY
YALE UNIVERSITY
February 1923

ON THE SPECTRUM OF NEUTRAL HELIUM

By C. V. RAMAN and A. S. GANESAN

ABSTRACT

Silberstein's formula for the lines of neutral helium.—As objections to the validity of the formula $\nu=4N\left(\frac{1}{n_1^2}-\frac{1}{m_1^2}+\frac{1}{n_2^2}-\frac{1}{m_2^2}\right)$ it is pointed out that the ionizing potential computed by means of this formula does not agree with that given by experiments; that no definite principle of selection has been given to indicate observable lines; that there is no arrangement of lines as regards series relationship, division into singlet and doublet systems, or of intensities. The difference between the actual and the calculated frequencies is shown to be the same as that to be expected on the assumption that the coincidences are purely accidental; and the same frequencies can be obtained with different sets of numbers.

In his paper on this subject contributed to the September number of the *Astrophysical Journal* (**56**, 119, 1922), Dr. Silberstein has suggested a formula for the explanation of the spectrum of neutral helium. This formula may be written as follows:

$$\nu=4N\left\{\left(\frac{1}{n_1^2}-\frac{1}{m_1^2}\right)+\left(\frac{1}{n_2^2}-\frac{1}{m_2^2}\right)\right\},$$

where n_1, n_2, m_1, m_2 are four independent integers such that $m_1>n_1$ and $m_2>n_2$. "This formula amounts, obviously, to putting $\nu=\nu_1+\nu_2$ where ν_1 and ν_2 are frequencies of any two lines of ionized helium, observed or only theoretical. This would then be a new kind of combination principle, the sum of the frequencies belonging to one atomic system (ionized helium) giving the frequency for another system (neutral helium)." With this formula Dr. Silberstein has succeeded in explaining 84 of the total 111 observed lines of the helium spectrum, to a very close approximation.

Now, the question is whether the formula has any physical basis or significance. Silberstein derives it on the assumption that "the mutual perturbation of the two electrons is practically nil or negligible," and in support of this says that "as a matter of fact we have no good evidence that the electrons, especially as trabants of the nucleus within the atom, do act upon each other at all, and

some bold modern physicist, encouraged by the recognized prohibition to radiate, might deny the electrons the right to interact while they are busy obeying the orders of the central body driving them around on stationary orbits."

But there are some very weighty objections against the formula and the results derived therefrom.

The ionization potential as calculated with this formula is quite different from that which is actually observed. The ionization potential is the work per unit charge required in moving an electron out of the influence of the atom. On the assumption that the two electrons do not act on each other, the ionization potential would obviously be

$$V=\frac{4\times 109723.2}{8102}=54.2 \text{ volts.}$$

But actually it is known that to doubly ionize the helium atom about 80 volts are necessary.

The formula, as it stands, represents a very large number of lines and there is absolutely no principle of selection followed which prima facie indicates the lines to be chosen and those to be rejected. As a matter of fact, the selection actually made appears to be more or less haphazard, the only aim being to choose such numbers as happen to give values coinciding most closely with the observed values. The want of a definite principle of selection is a strong objection against accepting the proposed formula as having any physical significance.

Again, as is well known, the observed lines of the helium spectrum fall into certain definite series which Silberstein dismisses as empirical, but which nevertheless have a definite physical meaning. Thus in each series as we pass from the less to the more refrangible part of the spectrum the lines become more and more crowded, grow fainter and fainter, and tend to converge to a definite limit. But Silberstein's formula shows no arrangement either with reference to series relationships or with reference to singlet and doublet systems of lines.

Nor does it suggest any arrangement with regard to the intensity of the lines. Bohr's theory, which has successfully explained the

spectra of hydrogen and ionized helium, suggests that the probability of electrons remaining in distant orbits is very small and that more electrons are to be found in orbits closer to the nucleus. And from the analogy with the spectra of hydrogen and ionized helium, we should expect that more intense lines are obtained by using comparatively smaller quantum numbers than for the less intense ones. In Silberstein's table of values some of the first lines of great intensities are not to be seen at all (probably no combination within the limits prescribed for m and n gives them), and of the rest, more frequently than not, the more intense lines are obtained by larger quantum numbers than the less intense ones. Table I gives some examples.

TABLE I

Observed ν	Intensity	Quantum Numbers, and Calculated ν
9231.86 9230.83	200 (P.d.)	Not included
25708.63	10 "	Not included
31361.12	8 "	$\left(\frac{16}{4} \cdot \frac{19}{8}\right)$ 31358.
33944.75	6 "	$\left(\frac{15}{4} \cdot \frac{30}{7}\right)$ 33950.
37537.5	1 "	$\left(\frac{4}{3} \cdot \frac{18}{5}\right)$ 37536.
37798.22	"	$\left(\frac{4}{3} \cdot \frac{20}{5}\right)$ 37794.
41150	5 (S.d.)	Not included
21211.35	3 "	$\left(\frac{6}{5} \cdot \frac{16}{5}\right)$ 21205.5
27374.48	1 "	$\left(\frac{6}{4} \cdot \frac{9}{5}\right)$ 27377.
4857.34	20 (S.s.)	$\left(\frac{10}{7} \cdot \frac{15}{4}\right)$ 4856.7
22528.65	1 "	$\left(\frac{7}{5} \cdot \frac{11}{5}\right)$ 22527.4

Many of the lines can be obtained by numbers other than those chosen by Silberstein (confined, of course, between his limits

$m \geqslant 32$ and $n \geqslant 9$), and in some cases these agree more closely with the observed values than those given by Silberstein. Thus to give a few examples:

TABLE II

Observed	Silberstein's Values		Other Values	
24830.....	$\left(\frac{13}{4} \cdot \frac{n}{n}\right)$	24834.	$\left(\frac{10}{11} \cdot \frac{11}{9}\right)$	24833.
26938.....	$\left(\frac{30}{4} \cdot \frac{n}{n}\right)$	26943.	$\left(\frac{16}{5} \cdot \frac{20}{6}\right)$	26935.4
28574.....	$\left(\frac{4}{3} \cdot \frac{16}{7}\right)$	28577.	$\left(\frac{11}{4} \cdot \frac{26}{9}\right)$	28572.8
36208.....	$\left(\frac{5}{4} \cdot \frac{20}{4}\right)$	36208.7	$\left(\frac{10}{4} \cdot \frac{10}{5}\right)$	36208.7
36592.....	$\left(\frac{5}{4} \cdot \frac{25}{4}\right)$	36603.	$\left(\frac{17}{4} \cdot \frac{17}{6}\right)$	36585.
			$\left(\frac{13}{4} \cdot \frac{32}{6}\right)$	36596.7
19824.....	$\left(\frac{5}{4} \cdot \frac{14}{6}\right)$	19827.	$\left(\frac{6}{\ } \cdot \frac{23}{9}\right)$	19828.
			$\left(\frac{14}{5} \cdot \frac{22}{9}\right)$	19828.
			$\left(\frac{28}{6} \cdot \frac{24}{7}\right)$	19827.
20358.....	$\left(\frac{5}{4} \cdot \frac{16}{6}\right)$	20352.	$\left(\frac{17}{5} \cdot \frac{20}{9}\right)$	20358.3
			$\left(\frac{11}{5} \cdot \frac{32}{8}\right)$	20357.6
27508.....	$\left(\frac{9}{4} \cdot \frac{18}{8}\right)$	27516.	$\left(\frac{11}{4} \cdot \frac{16}{9}\right)$	27507.6

The average difference between the actual frequencies of the helium lines and the values nearest to them given by Silberstein's formula is exactly what we should expect from the simple theory of probability on assuming the coincidences to be fortuitous. Thus for instance, if $m \geqslant 32$ and $n \geqslant 9$, the total number of lines obtained with Silberstein's formula, lying between $\nu = 28000$ and $\nu = 29000$,

is found to be 161. If these lines were evenly distributed between these limits, the interval between successive lines would be $\frac{1000}{161} = 6.1$ and the maximum difference of any value of ν between 28000 and 29000, chosen at random, and the nearest number in the list would be 3.0. The minimum error being 0, the average error would be 1.5. Taking the 19 observed lines of helium between these limits and finding for each the nearest number from the list, the average actual error is found to be 2.6; that is, actually greater than the number indicated on the assumption that the coincidences are purely fortuitous. This is what we should expect; for, owing to the non-even distribution of the numbers, there are certain gaps with differences of about 20 to 25, and some 3 of the 19 lines falling in these gaps eventually increase the average error. From Silberstein's table the average error for the 15 lines which he has given between these limits is about 3.5. This is due to the fact that there are combinations other than Silberstein's that are nearer the observed values.

These facts clearly show that coincidences between the actual frequencies and those given by Silberstein's formula must all be regarded as purely fortuitous and having no physical significance whatever.

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ON THE SPECTRUM OF HELIUM, A REJOINDER TO C. V. RAMAN AND A. S. GANESAN

By LUDWIK SILBERSTEIN

ABSTRACT

Consideration of objections to the Silberstein formula for the spectrum of neutral helium. (1) The author disclaims ignoring the fact that the observed lines fall into certain *definite series*, and points out that the formula covers twenty-three of the thirty so-called combination lines. (2) A discussion of the *ionization potential* was purposely omitted from his original paper since the mutual relation of the electrons in the normal state of the atom is still an open question. He has preferred to deal with it in a separate paper. (3) In order to show that *the coincidences of lines are not fortuitous* he computes that for $3 \leqq n \leqq 8$ and $4 \leqq m \leqq 20$ the theoretical number of lines in the interval $19{,}800 \leqq \nu \leqq 37{,}800$ is 631. Of these, 44 coincide with observed lines, of which there are 96 in this interval. The probability that these coincidences are accidental is of the order 10^{-13}.

The substance of the objections raised against the spectrum formula

$$\nu = 4N\left(\frac{1}{n_1^2}+\frac{1}{n_2^2}-\frac{1}{m_1^2}-\frac{1}{m_2^2}\right) \equiv 4N\left(\frac{m_1}{n_1}\cdot\frac{m_2}{n_2}\right) \qquad (1)$$

by Raman and Ganesan in this issue of the *Astrophysical Journal* has already been given in Professor Raman's letter published in *Nature* of November 25 (**110,** 700, 1922), to which I replied at some length in *Nature* of January 13 (**111,** 46, 1923). The full paper, moreover, upon which the last said letter was based, was read at the Boston meeting of the American Physical Society and is now in the course of publication in the *Philosophical Magazine.* It contains all numerical details relevant to the question. Under these circumstances it will be enough to make here but a few concise remarks, the more so, as my reply in *Nature* could not possibly have reached Calcutta before Raman and Ganesan wrote their note which arrived at Chicago in the middle of January.

In the first place then I have never "dismissed as empirical" the fact that the observed lines "fall into certain definite series." On the contrary, I have emphatically expressed the regret for seeing the series *lacerated* through the proposed representation. At the same time, however, I drew attention to some regularities.

I should like to add that with the thirty so-called "combination lines," of which my formula covers twenty-three, no such claims or objections can be associated. Many of these lines can scarcely be said to be represented at all by the empirical "combinations," eight of these showing deviations of 2 to 3 and one even of 7.2 A (cf. Fowler's *Report*, p. 94), whereas they are covered by my formula with particular ease, as for instance the three *C.d.* lines, all represented by $\left(\frac{5 \cdot m_2}{4 \cdot 4}\right)$.

With regard to the ionization potential of the (normal) helium atom, I have purposely abstained from dealing with it, since there was no occasion of descending to one-quantic orbits, the majority of my cases concerning $n \geq 4$. Essentially relevant for the ionization work is the behavior of the intra-atomic electrons within the region of the few first quantum numbers, so that the actual applications made of my formula do not by any means exclude a treatment of the ionization problem based on appropriate electronic interaction. The configuration of the electrons in the normal helium atom, even on Coulomb lines, is still an open question (and a very hard one), and may here be left alone. I deal with it in another paper which, however, has nothing in common with the present discussion. In other words, a formula may be successful in the region $\nu = 17{,}000$ to 38,000, without pretending to cover the extreme ultra-violet Lyman series, of which λ 504 or 506 A (the equivalent of the ionization potential of 24.5 volts) is the limit.

The most crushing contention of the opponents of my formula, however, and the easiest to repel, seems to be, that the coincidences given by it "must all be regarded as purely fortuitous" and therefore as "having no physical significance whatever." The latter verdict of Raman and Ganesan had better be left alone, as it would scarcely be feasible to discuss here the subtle concept of "physical significance" and the still harder question, whether a given set of conceptual results does or does not possess it. It is enough to repel the imputation of fortuitousness, and since this has been most particularly dealt with in the aforesaid letter and paper, it will suffice to point out the reason for the discrepancy of opinion held in this respect by the writer and by his two opponents. Raman

and Ganesan obtain with my formula such large numbers of theoretical lines (as 161 between $\nu = 28{,}000$ and 29,000) and thence also such a high estimate of the probability of chance coincidences, because they insist, very unjustly, upon pushing m as high as 32 (and n up to 9), although such high numbers were purposely avoided, the table in my first paper containing "32" only once, and "31" not at all. The significant fact, and one that speaks very strongly in favor of the proposed formula, is that if we limit the quantum numbers to the intervals

$$3 \leqq n \leqq 8 \quad \text{and} \quad 4 \leqq m \leqq 20, \tag{2}$$

the number of theoretical lines is cut down very considerably and yet a large number of good coincidences still remains. One such example is mentioned, and estimated for its probability, in *Nature* of January 13. Another, still more convincing example (sacrificing only one out of 45 lines) is this:

The number of distinct theoretical lines satisfying the conditions (2) and falling within the interval

$$\nu = 19{,}800 \text{ to } 37{,}800 \tag{3}$$

is 631 only. Of these as many as 44 coincide with observed lines, whose total in the interval (3) is 96. The mean deviation is $|\delta\nu| = 2.57$. Now, a straightforward computation will show that the probability of such an event considered as a fortuitous set of coincidences is, in round figures,

$$P_{44} = 1 - \Theta(7), \tag{4}$$

where Θ is the error-function. This formula speaks for itself. In fact, for $x = 4.80$, the largest argument one has ever encountered in error-function tables, $1 - \Theta(x)$ is just of the order 10^{-11}. For $x = 5.30$ it is about 20 times smaller, and for $x = 7$, as in our case, it is well below 10^{-13}. In fine, the "fortuitous" hitting of these 44 lines under the said circumstances is as improbable as making forty "heads" in succession in tossing a coin.

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SPECTROSCOPIC OBSERVATIONS OF BOTH COMPONENTS OF THE VARIABLE DOUBLE STAR X OPHIUCHI[1]

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ABSTRACT

Spectra and magnitudes of both components of X Ophiuchi.—This double star of separation 0".22 consists of a typical long-period variable of class M6e of apparent mag. 6.8 to about 12, and a star of class Ko of mag. 8.9. The former can be observed spectroscopically at maximum, the latter at minimum. Twenty spectrograms have been secured of which six show the K-type spectrum. The absolute magnitude of the K-type star as determined from these spectrograms by Mr. Adams is 2.4; from this it follows that the absolute magnitude of the variable at maximum is 0.3.

Radial velocity of both components.—The radial velocities derived from the dark lines of the M- and K-type components are −70.6 and −70.8 km, respectively, while the bright lines of the M star yield a mean value of −83.4 km. It is probable that the displacements of the dark lines of the variable and of Me stars in general give essentially the star's radial velocity. Near maximum the wave-lengths of the bright lines are shortened by about 0.2 A, by motions within the star or by other causes, and for several weeks after maximum are algebraically smaller than at other times. This effect is probably typical of variables of class Me.

X Ophiuchi presents the only known instance in which a long-period variable is one component of a visual double star. The duplicity of this star was discovered by Hussey[2] in 1900. His observations of position angle and distance were 195°.2 and 0".22, respectively. Remeasurement by Van Biesbroeck[3] twenty years later showed the position angle to have decreased by 24°, the separation being the same. He decided that the northern component is the variable, and Gingrich[4] has confirmed this by his measurements of parallax photographs.

It will obviously be many years before the orbital elements are accurately known, but as a first approximation we may assume a circular orbit whose plane is perpendicular to the line of sight. The radius of the relative orbit would then be 0".22, and the period 300 years. Using the spectroscopic parallax,[5] 0".005, determined by Mr. Adams (see p. 252), the distance between the stars is calculated to be forty-four astronomical units, and the combined mass 0.95 that

[1] *Contributions from the Mount Wilson Observatory*, No. 261.

[2] *Astronomical Journal*, **21**, 35, 1900. [3] *Popular Astronomy*, **29**, 278, 1921.

[4] *Mt. Wilson Contr.*, No. 238; *Astrophysical Journal*, **56**, 132, 1922.

[5] Measurement of the trigonometric parallax by Van Maanen yielded the value 0".000. *Publications Astronomical Society of the Pacific*, **33**, 319, 1921.

of the sun. If each of the stars has one-half of the combined mass, the orbital speed of each with respect to the center of mass is 2.2 km. These values are, of course, subject to large corrections.

The period of the light variations is 337 days, which is about the average for long-period variables. The mean range in brightness is, however, only 2.1 mag., which is considerably less than usual for variables of this type; but this refers to the combined magnitude of the two components. The range of the variable component is considerably greater, probably about 5 magnitudes. The spectrum at maximum is class M6e, but at minimum this spectrum is overpowered by that of the southern component, which is of class Ko. Six spectrograms (see Table II), taken near minimum, show the Ko spectrum fairly free from interference by that of the variable star. From these Mr. Adams has had the kindness to determine the absolute magnitude, finding it to be +2.4. The apparent magnitude is 8.9, and the spectroscopic parallax 0".005. The available photometric data may be combined to yield the results indicated in Table I.

TABLE I

MAGNITUDES OF THE COMPONENTS OF X OPHIUCHI

COMPONENT	MAXIMUM			MINIMUM		
	Vis.	Ph.	Abs. Vis.	Vis.	Ph.	Abs. Vis.
Combined.....	6.7	8.3	0.2	8.8	9.9	2.3
Northern.....	6.8	8.6	0.3	12	14	6
Southern.....	8.9	9.9	2.4	8.9	9.9	2.4

Thus it happens that at maximum the K-type component contributes only one-tenth of a magnitude to the combined light, while at minimum it preponderates so strongly that this relationship is reversed and the *variable* contributes only about the same proportion. When the combined magnitude is brighter than 7.5, the M-type spectrum is recorded in the Hγ region with very little interference from the other component. On the other hand, when the combined light is fainter than 8.0 mag., the K-type spectrum appears quite pure in this region, although near Hβ the bands of the M-type star show plainly until magnitude 8.6 or 8.8 is reached. Assuming color indices of 1.0 and 1.8 mag.[1] for the K- and M-type stars, respec-

[1] A mean value used by the Harvard College Observatory for Me variables. It may vary during the light-cycle.

tively, as in Table I, we find that at combined magnitude 7.5 the M-type star is 1.1 mag. brighter visually, and 0.3 mag. photographically than the K-type component; when the combined magnitude is 8.0, the M-star is 0.3 brighter visually and 0.5 fainter photographically than the other. Thus it appears that the spectrum of either star may be obtained practically pure, if it is a half-magnitude brighter photographically than the other.

This star offers a unique opportunity to derive a value of the absolute magnitude of a long-period variable by a direct method, through comparison with the K-type component whose absolute magnitude can be determined spectroscopically in the usual way. The distances of these variables are, in general, so great that trigonometric parallaxes can scarcely be used except in a statistical manner. The high absolute magnitude, 0.3 at maximum, found for the variable component of X Ophiuchi, is close to the mean value for giant M stars; and since the variable component appears to be a typical long-period variable, the presumption is that long-period variables in general are giant stars. This is borne out by the fact that the mean absolute magnitude of these stars derived by several statistical investigations is about the same as that found for X Ophiuchi. Discussions of the causes of variability and of the evolutionary relationships of these stars must take into account their high luminosity at maximum.

Observations at the Harvard College Observatory,[1] at Ann Arbor,[2] and at Mount Wilson have shown the spectrum to be of class M at maximum. In the spectral classes Mo–M10, which are to replace the subdivisions Ma–Mc,[3] the class of X Ophiuchi would be M6e. Photographs of the spectrum have been secured here at times fairly well distributed over the light-curve. The spectrum shows striking changes, aside from the effect of the K-type component, which correspond to those regularly exhibited by long-period variables.[4] Hence the northern component of this double star may be regarded as a typical long-period variable.

[1] Henry Draper Catalogue, *Harvard Annals*, **97**, 258, 1922.

[2] *Publications of the Observatory, University of Michigan*, **2**, 58, 1916.

[3] *Transactions of the International Astronomical Union*, **1**, 97, 1922.

[4] *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 185, 1921.

The Mount Wilson spectrograms are listed in Table II. The first three columns give, respectively, the date, visual magnitude, and the number of days before (−), or after (+) maximum. I am indebted to Mr. Leon Campbell, of the Harvard College Observatory, for the photometric data. The fourth and fifth columns give the velocities derived from the absorption lines of the M- and K-type components, respectively, with the number of lines on which each determination is based. The last column gives the velocity derived from the emission lines, and the lines used on each plate.

TABLE II

RADIAL VELOCITIES OF X OPHIUCHI

DATE	MAG.	PHASE	VELOCITY		
			M	K	E
1919 June 9*	6.6	− 11	.....	.....	−87.7 γδ
June 10*	6.6	− 10	.....	.....	−84.7 γδ
1920 Apr. 8	7.3	− 53	.....	.....	−73.9 γδ
May 1	6.9	− 30	−69.5 12	.....	−76.9 γδ
May 4*	6.8	− 27	−68.1 12	.....	−78.6 γδ
May 31	6.6	0	−69.9 34	.....	−83.2 γδ 4202
July 5*	7.0	+ 35	.....	.....	−86.9 γδ 4202
July 6	7.0	+ 36	−70.0 7	.....	−83.1 γδ 4202
Sept. 2	8.3	+ 94	.....	−69.3 10	−78.6 γδ 4202, 4308, 4571
Sept. 27	8.8	+119	.....	.....	−75.2 4202, 4308, 4571
1921 Apr. 26	7.0	− 20	−72.7 10	.....	−83.5 γδ
Apr. 26†	7.0	− 20	.....	.....	−84.6 γδ
May 24*	6.6	+ 8	−70.4 19	.....	−89.3 γδ
June 22*	7.3	+ 37	−73.3 24	.....	−88.0 γδ 4202
Aug. 11	8.2	+ 87	.....	−72.9 17	−89.6 γδ 4202, 4308, 4571
Sept. 21	8.2	+123	.....	−73.2 18	.....
Oct. 11	8.4	+143	.....	−73.7 18	.....
1922 June 14*	7.6	+ 60	.....	.....	−86.0 γδ 4202, 4571
Aug. 9	8.6	+116	.....	−64.6 14	(−65.) 4202, 4308, 4571
Sept. 8	8.7	+146	.....	−71.2 13	(−82.) 4308
Mean	.....	.....	−70.6	−70.8	−83.4

* Sixty-inch telescope. † Three-prism spectrograph.

With a single exception the plates were made with one-prism spectrographs attached to the 60-inch or the 100-inch telescope; the dispersion at Hγ is 36 A per mm. The second plate, of April 26, 1921, was secured with a three-prism spectrograph belonging to the Lowell Observatory. This was being used for a special investigation in connection with the Hooker telescope by Dr. V. M. Slipher, and it was through his courtesy that the exposure on X Ophiuchi was made. The dispersion at Hγ with this instrument is 12.5 A per mm. On this plate the bright Hγ and Hδ lines are narrow and fairly well defined, but they are not quite so sharp as

the comparison lines of the iron arc. The measured velocity agrees with that derived from the one-prism spectrograms, and gives a desirable check on their accuracy. Measurements of the emission lines on the last two plates were rendered uncertain by the presence of the strong K-type spectrum. The values derived from them were not used in forming the mean.

The discrepancy of 12.8 km between the apparent velocities from the bright and dark lines of the M-type star corresponds in sign and magnitude to similar displacements observed in the spectrum of *o* Ceti and of many other Me variables. It is of great interest to find that the velocity from the K-type star is practically identical with that from the absorption lines of the M-type star. Since the orbital speed is probably about 2.2 km and the inclination small, the relative radial velocity of the two components must be very nearly zero. Hence we are led to the conclusion that the result derived from the dark lines of the M-type spectrum represents essentially the radial velocity of the star, and that the bright lines are displaced toward the violet by some unknown cause. It is probably safe to generalize this statement and make it apply to all variable stars of class Me in which the usual relative displacement of bright and dark lines exists. Support is lent to this assumption by the fact that, in a solution for solar motion based on the radial velocities derived from the bright lines of 83 variables, the K-term was found to be −12 km.[1]

The apparent velocities from the emission lines of X Ophiuchi are plotted against the light phase in Figure 1. The observations extend in phase from −53 days to +119 days, covering about one-half of the period. The velocities are not constant, but appear to have low values for a month or two after maximum light. Of the Ann Arbor spectrograms, one taken in 1914 at a phase of +24 days gave a velocity of −91 km, and three others taken in 1915 gave a normal place of −84.6 km at −69 days. Thus they exhibit the same effect as the Mount Wilson spectrograms, although to a lesser degree. A close quantitative agreement is perhaps not to be expected, especially as the record shows that the Ann Arbor velocities have rather low weight. In view of the lack of regularity in the light-curve, it would not be surprising if different cycles

[1] *Popular Astronomy*, **29**, 637, 1921.

yielded somewhat different velocity effects. Three other stars, namely, R Leonis, χ Cygni, and T Cephei, seem to exhibit the same effect, and fragmentary observations indicate about the same behavior by a half-dozen other variables. It thus seems to be typical of Me variables that the bright lines have slightly shorter effective

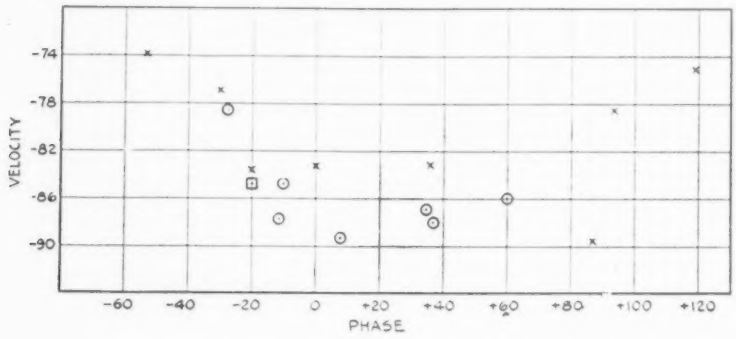

FIG. 1.—Phase in days counted from maximum light, and apparent velocities (km per sec.) derived from emission lines. ×, single-prism spectrograph attached to the 100-inch telescope; ⊙, single-prism spectrograph attached to the 60-inch telescope; □, three-prism spectrograph attached to the 100-inch telescope.

wave-lengths during several weeks after maximum than at other times.[1] This is about the time of the maximum relative intensity of the hydrogen lines.[2]

If the displacements of the bright hydrogen lines are due to an outflow of incandescent hydrogen, it follows that on first coming into view after minimum the gas has a low velocity, and is subject to acceleration while under observation, or is replaced by other hydrogen having a higher outward velocity. This would indicate the existence of agencies acting during an appreciable fraction of the light-period, rather than a sudden outburst followed by a gradual resumption of normal conditions.

MOUNT WILSON OBSERVATORY
March 1923

[1] Errors of measurement and the inclusion, during the declining phase, of other bright lines than those due to hydrogen do not seem adequate to explain the observed effects. These questions will, however, be considered in another contribution.

[2] *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 185, 1921.